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PROBLEMS OF STAR FORMATION

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ABSTRACT

The lectures deal mainly with the condensation of self-gravitating masses of gas from cosmic clouds, with special attention to difficulties due to angular momentum and a magnetic field. Emphasis is laid on the dynamical problems of the early stages, before opacity becomes important.

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63

1. GRAVITATIONAL INSTABILITY IN A NON-ROTATING NON-MAGNETIC MEDIUM

1.1 Jeans's classical treatment:

Plane pressure waves are imagined propagating in a self-gravitating uniform gas at rest. If the disturbance is $\propto \exp[i(kx-\omega t)]$, there results a simple dispersion relation between the angular frequency ω and the wave-number k :

$$\omega^2 = k^2 c^2 - 4\pi G \rho_0 , \tag{1}$$

where c is the velocity of sound when gravity can be ignored, ρ_0 is the unperturbed density, and G the gravitational constant. Thus there exists a critical wave number

$$k_J = \left[\frac{4\pi G \rho_0}{c^2} \right]^{1/2} . \tag{2}$$

If $k \gg k_J$ — small wavelengths — $\omega/k \simeq c$: the disturbances are just sound waves, slightly modified by the gravitational reduction of the elasticity of the medium. For $k < k_J$, ω/c is imaginary; the density perturbations are not reversed by gas pressure but are amplified by self-gravitation, the e-folding time being

$$\frac{1}{|\omega|} = \frac{(4\pi G\rho_0)^{-1/2}}{(1 - k^2c^2/4\pi G\rho_0)^{1/2}} = \frac{(4\pi G\rho_0)^{-1/2}}{[1-(k/k_J)^2]} . \tag{3}$$

We note that the time is infinite for $k = k_J$, and decreases monotonically with increasing wavelength to the limit $(4\pi G\rho_0)^{-1/2}$. The minimum unstable length λ_J is one half a wavelength, i.e.

$$\lambda_J = \frac{\pi}{k_J} = \left(\frac{\pi}{4} \frac{c^2}{G\rho_0} \right)^{1/2} . \tag{4}$$

As a numerical example, consider an HI medium with $\rho_0 = 10\, m_H$, and temperature $\Theta = 10^2\,°K$; then $\lambda_J \simeq 25$ parsecs, and the shortest e-folding time is $\simeq 10^7$ years. As the work of Spitzer and Savedoff[2] yields a much shorter time - about 100 years - for radiative absorption and emission to iron out temperature variations. the isothermal rather than the adiabatic sound speed must be used in (3) and (4).

The main objection to Jeans's treatment is that insufficient attention is paid to the zero-order state which is being perturbed. The infinite uniform medium cannot have a gravitational field, as there is no direction in which it can act; yet Poisson's equation demands that there be a gravitational field with a non-vanishing divergence. But if, more realistically, one considers a finite, roughly spherical cloud, in equilibrium under its own thermal pressure, then from the virial theorem (see later),

$$T = -\tfrac{1}{2}\Omega \simeq \frac{GM^2}{2R}, \tag{5}$$

where the total kinetic energy $T \simeq Mc^2$, Ω is the gravitational energy, and M and R are the mass and radius of the cloud. Thus

$$c^2 \simeq \frac{GM}{2R} \simeq \frac{2\pi}{3} G\bar{\rho} R^2, \tag{6}$$

$\bar{\rho}$ being the mean density; and the critical length (4) is

$$\left(\frac{\pi}{4}\frac{c^2}{G\bar{\rho}}\right)^{1/2} \simeq R. \tag{7}$$

Clearly, then, one cannot apply the Jeans analysis, which assumes a uniform zero-order density, to disturbances of wavelength comparable with the background scale-height. The analysis is valid only for wavelengths much less than the radius, yielding just sound-waves slightly modified by self-gravitation, but no instability. If, on the other hand, the temperature and density are such that the Jeans length is much less than the radius, then the cloud must be contracting, and the instability problem must be reconsidered. Jeans himself was aware of this difficulty: he admits that his treatment is relevant only if the gas cools in a time much shorter than the gravitational free-fall time. Such a possibility has been envisaged recently by Hoyle, in a discussion of galaxy formation. Otherwise, we are forced to admit the overall collapse of the gas cloud, and study the instability problem against a freely-falling background.

1.2 The initial collapse of a gravitating cloud.

The following treatment is due to Ebert[3], Bonnor[4] and McCrea[5]. Consider a cloud subject to an external pressure p. The virial theorem (first derived by Poincaré) states

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2T + \int_S p\,\underset{\sim}{r} \cdot \underset{\sim}{dS} + \int_V \rho\,\underset{\sim}{r} \cdot \underset{\sim}{g}\, dV, \tag{8}$$

where

$\underset{\sim}{g}$ = gravitational acceleration,

$\underset{\sim}{dS}$ = surface element with an inward-directed normal,

V = volume of cloud,

I = $\int_V \rho r^2 dV$ = "moment of inertia about the origin,"

T = kinetic energy
= sum of thermal, turbulent and bulk motion energies.

The theorem is derived by taking the scalar product of $\underset{\sim}{r}$ with the equation of motion of each particle, and then summing. The forces due to collisions between particles cancel except for those between surface particles and the external matter, which are absorbed into the pressure p; magnetic forces are temporarily ignored.

In equilibrium $dI/dt = 0$; for equilibrium to persist $d^2 I/dt^2 = 0$, so that a necessary (but by no means a sufficient) condition for equilibrium is

$$pV = \frac{1}{3}\Omega + \frac{2T}{3}. \tag{9}$$

Here p has been assumed uniform and the surface integral transformed by Gauss's theorem; and

$$\Omega = \int \rho\,\underset{\sim}{r} \cdot \underset{\sim}{g}\, dV \quad = \text{gravitational energy}. \tag{10}$$

It is clear that in a steady state any macroscopic kinetic energy must be turbulent or rotatory. If T is purely thermal, and the gas is monatomic,

$$T = \text{internal energy } U = \frac{3}{2}\frac{R\Theta M}{\mu} = \frac{3}{2}Mc^2,$$

where Θ is absolute temperature and μ the mean molecular weight. Bonnor's "modified Boyle's law" is then

$$p = c^2\bar{\rho} - \frac{1}{3}\left(-\frac{\Omega}{V}\right) \qquad (-\Omega > 0). \tag{11}$$

Now consider p to be steadily increased, isothermality being maintained by radiative loss of energy of compression. Then

$$c^2 \bar{\rho} \propto 1/R^3, \tag{12}$$

and

$$-\frac{\Omega}{V} \simeq \frac{GM^2}{R\left(\frac{4\pi}{3}R^3\right)} \propto \frac{1}{R^4} \tag{13}$$

Hence there exists a maximum p_c to the external pressure which the cloud can withstand, after which gravitational collapse starts. The critical radius R_c is necessarily somewhat larger - by a factor two - than the critical radius given by applying the virial theorem without the external pressure (equation 6). For $p < p_c$, there exist also equilibria with $R < R_c$, but they are unstable and so of no interest.

The importance of keeping in the external pressure term is not because of the change in the critical radius, but because it keeps the problem somewhat closer to reality. The exact solution of the Emden equation to the isothermal gravitating gas sphere does not yield a zero density at a finite distance, so that an external pressure is required to keep a cloud of finite mass in equilibrium. Further, current ideas on cloud formation require an external compression - e.g.from an expanding $H\underline{\overline{II}}$ region within a galaxy, or from hot intergalactic gas on the cosmological level. A violent enough compression will bring about gravitational collapse; otherwise the cloud will re-expand and try and reach pressure equilibrium with the low density, hot, inter-cloud gas.

The effect of a strong centrifugal field will be considered in Section 2.1. A turbulent field will also contribute to pressure balance, though in the absence of an energy supply to keep the cloud stirred up - e.g. a hot star - the turbulence must decay, its energy being thermalised and radiated away. In fact if the turbulent pressure is to exceed the thermal, the velocities must be supersonic, so that rapid dissipation in shocks is to be expected. This seems to be supported by measurements of line-widths, which indicate subsonic random velocities within individual clouds, (as compared with the supersonic velocities of clouds relative to each other). Thus turbulence should not alter the order of magnitude of the conclusions. Its importance is rather that it necessarily provides a field of density fluctuations which are essential to the break-up of an isothermal contracting cloud into stars - see Section 1.4.

So far we have assumed a fixed, uniform temperature for the cloud. A fixed external pressure can become critical and so cause collapse if the cloud systematically cools below its initial temperature - e.g. because of a sharp increase in the fraction of hydrogen in molecular form, itself brought about by the initial compression. It is also possible that such cooling occurs locally, so allowing further compression and ultimate gravitational collapse of a mass of stellar order, while the cloud as a whole stays in equilibrium. However, this seems a rather exceptional case, as compared with the break-up of an isothermal cloud that is itself collapsing.

1.3 Collapse without fragmentation.

Suppose now that the critical pressure is exceeded, and the cloud contracts. The compression of the gas generates heat, which must be radiated if isothermality is to be maintained. Thus in a diffuse cloud of pure monatomic hydrogen, collisional ionization and radiative recombination start near $10^4\,{}^\circ K$ and keep the cloud temperature nearly constant at this value. In a cool HI cloud, dust grains and H_2 molecules can radiate the heat of compression without the temperature rising much above the standard value of about $100^\circ K$, as long as the opacity is low.[2] Then once contraction starts it soon becomes gravitational free-fall because the thermal energy stays constant while the gravitational energy increases in absolute value.

Once pressure can be ignored, the equation to the collapse of a spherically symmetric mass is

$$\ddot{r} = -\frac{Gm}{r^2}. \tag{14}$$

Here $r(m,t)$ is the radius of the mass sphere m at time t — m is used as an independent variable. Taking the special case in which $\partial r/\partial t = 0$ at $t = 0$ for each m, we find the solution of (14) to be

$$r = r_i(m) \cos^2 \bar{\theta},$$
$$\bar{\theta} + \tfrac{1}{2} \sin 2\bar{\theta} = t \left[\frac{8\pi}{3} G \bar{\rho}_i(m) \right]^{1/2}, \tag{15}$$

where

$$r_0(m) = r(m, 0),$$

and

$$\bar{\rho}_0(m) = \frac{m}{\frac{4\pi}{3} r_0^3} = \text{initial mean density}$$

Thus the mass sphere m collapses in a characteristic time

$$\tau = \left[\frac{3\pi}{32 G \bar{\rho}_0(m)}\right]^{1/2}: \qquad (16)$$

inevitably of the same order as the shortest Jeans e-folding time. The gravitational energy released all becomes (in this approximation) kinetic energy of inward bulk motion.. In fact, even when the pressure gradient is dynamically negligible, there is always some heat generated: the free-fall approximation depends on this not being able to build up a strong pressure gradient. As the density goes up, the opacity increases and the cloud becomes optically thick in the frequencies radiated. Something like the Planck radiation density for these frequencies in then built up, and some of the gravitational energy released builds up a temperature gradient adequate to drive out the heat of compression. Provided the increased thermal energy is still small compared with the gravitational the free-fall continues.

At still greater densities the opacity becomes so great that collapse at the free-fall rate is essentially adiabatic. Suppose that at a radius $R_1 \ll$ the initial radius R_0 the radiative loss is effectively cut off. In collapsing from R_0 to R_1 the cloud releases gravitational energy

$$\Omega_0 - \Omega_1 \simeq -\frac{GM^2}{R_0} - \left(-\frac{GM^2}{R_1}\right) = GM^2\left(\frac{1}{R_1} - \frac{1}{R_0}\right), \qquad (17)$$

the bulk of it becoming macroscopic kinetic energy T_1 and a negligible fraction being thermally dissipated and radiated away. As by hypothesis collapse beyond R_1 is adiabatic, it will be halted if there exists a radius $R_2 < R_1$ at which the internal energy. built up by the adiabatic collapse, has absorbed both T_1 and the further release of gravitational energy $(\Omega_1 - \Omega_2)$: i.e. the internal energy at R_2 is:

$$u_2 \simeq (\Omega_0 - \Omega_1) + (\Omega_1 - \Omega_2) = \Omega_0 - \Omega_2. \qquad (18)$$

However, for R_2 to be an equilibrium radius, the thermal part T_{th} of the internal energy must satisfy the virial theorem

$$2T_{th} + \Omega = 0 \tag{19}$$

(the surface pressure contribution being clearly negligible at these much higher densities). As usual

$$T_{th} = \frac{3}{2}(\gamma - 1)\,U \qquad \gamma = \frac{c_p}{c_v}, \tag{20}$$

so that in equilibrium

$$3(\gamma-1)\,U + \Omega = 0. \tag{21}$$

Thus the actual internal energy U_2 given by (18), exceeds the value for dynamical equilibrium, given by (21), if

$$\Omega_0 - \Omega_2 > -\Omega_2 / 3(\gamma - 1),$$

or

$$\gamma > \frac{4}{3} + (\gamma-1)(\Omega_0/\Omega_2). \tag{22}$$

(If adiabacy takes over only when $R_1 \ll R_0$ - so that $\Omega_0/\Omega_2 \ll 1$ - condition (22) is effectively $\gamma > 4/3$.) The cloud has then too much energy to stay in dynamical equilibrium at R_2, and so it bounces back. As the kinetic energy of oscillation is dissipated - e.g. in shocks - the cloud tries to approach equilibrium at a radius R', given by (21) with $U' = \Omega_c - \Omega'$

$$\Omega' = \frac{3(\gamma-1)}{(3\gamma-4)}\,\Omega_0. \tag{23}$$

With γ not too close to $\frac{4}{3}$ this radius is of the same order as R_o; e.g. with $\gamma = 5/3$ it is $R_o/2$, and is therefore likely to be greater than R_1.
Thus the equilibrium radius - given by assuming that all the gravitational energy released is thermally dissipated but <u>not radiated away</u> - yields a radius at which the cloud is not opaque; much of the thermal energy required to hold the cloud up would be immediately radiated away, and the cloud would collapse again, but with less energy than before. Clearly, (still on the assumption that the cloud does not break up into sub-condensations), dynamical equilibrium will be attained only when so much of the released gravitational energy has been dissipated and <u>radiated away</u>, that the equilibrium state is dense and therefore opaque: i.e. at a radius less than R_1. Subsequent contraction takes place with the cloud in hydrostatic equilibrium, at a rate determined by the energy leak down the equilibrium temperature gradient — Kelvin - Helmholtz contraction. The gravitational energy released supplies the surface loss and heats up the cloud to the value required for equilibrium.

If $\gamma < 4/3$ the internal degrees of freedom of the gas absorb too much of the released gravitational energy for the collapse to be halted (cf. Eddington[6]): the cloud is never able to transform its inward kinetic energy into internal energy, as required if the contraction is to be reversed at R_2. As an example, consider a cool dense opaque HI cloud, contracting slowly because of the strong absorption in the infra-red by molecules and dust grains. As the temperature slowly rises, molecules dissociate and dust grains volatilize, so reducing the opacity; and near 10^4 °K ionization of hydrogen begins, forcing γ down to near unity. Thus even if the cloud stays opaque the absorption of energy by collisional ionization will temporarily allow the cloud to fall freely.

<u>1.4 Fragmentation.</u>

A crucial part of the argument of Section 1.3 is the assumption that the cloud contracts or expands as a whole, without breaking up into sub-units, so that it is reasonable to talk of oscillations of a gas sphere, and, in particular, to expect kinetic energy to be rapidly dissipated in shocks. But the mass of a gravitationally bound galactic cloud is far above stellar order: for example, with $\Theta \simeq 10^2$ °K and $\rho \sim 10^3 m_H$ — a rather dense cloud - the critical mass for collapse is $\gtrsim 250 ☉$. Although it is possible to imagine extreme circumstances in which an interstellar globule of stellar mass is compressed until it becomes gravitationally unstable, it is far more plausible that most stars are formed in clusters by the fragmentation of massive gravitationally bound clouds of initially low density. Subsequent slow disintegration of star clusters feeds the general star field. The much greater mass of globular clusters as compared with galactic clusters suggests that the primeval galactic gas had a much higher temperature - say $10^4\ {}^\circ K$ — than the 100° or so estimated for present-day clouds, so that only very large masses were able to contract and form a star cluster.

If fragmentation could not occur, then we should have the problem of the ultimate fate of a gravitationally bound cloud of mass 250 $\odot$ or more that continually contracts, heating itself up and radiating excess energy. As it is, we want to find a dynamical description of the formation of sub-condensations; our object is to produce from the cloud a "gas" of blobs of small collision cross-sections, maintaining their spatial distribution because of their kinetic energy. By break-up into blobs that interact elastically, the natural tendency of the system to dissipate its energy is sharply cut off.

Hoyle first pointed out that under <u>isothermal</u> collapse the condition of gravitational binding holds for progressively smaller masses. Thus if initially

$$G \rho_0 R_0^2 \simeq c^2 \qquad (24)$$

so that gravitational collapse is just possible, then since $\rho \propto 1/R^3$ and with c constant, gravitation becomes dominant as R decreases, as already noted. Thus when the whole cloud has radius R, $\rho \propto \rho_0 R_0^3 / R^3$, and a blob of radius R' can be gravitationally bound if

$$G \frac{\rho_0 R_0^3}{R^3} R'^2 \simeq c^2 \simeq G \rho_0 R_0^2$$

or

$$R'^2 \simeq \frac{R^3}{R_0} ; \qquad (25)$$

and the mass of such a blob is

$$M' \simeq \rho R'^3 = \frac{\rho_c R_c^3}{R^3} \frac{R^{9/2}}{R_c^{3/2}} \simeq M \left(\frac{R}{R_c}\right)^{3/2}, \qquad (26)$$

where M is the cloud mass. Thus if $R/R_c \simeq 1/3$, $M'/M \simeq 1/5$ and $R'/R \simeq 1/\sqrt{3}$ — the cloud "can" break up into five gravitationally bound masses once its density has gone up by about 27. Hoyle shows that during the contraction of the cloud by about a factor 1/3, the near balance between gravitation and pressure ensures that most of the gravitational energy released is dissipated thermally and radiated away; but subsequent collapse of the cloud as a whole would generate a kinetic field, which if not dissipated would be strong enough to re-expand the cloud - essentially as in Section 1.3. But the formation of sub-condensations - each with an internal pressure field initially comparable with its self-gravitation - enables much more energy to be dissipated. Hoyle thus pictures a hierarchy: as each fragment collapses isothermally, sub-condensations form within it. The process is halted when the opacity is too high for the isothermal approximation to be valid, and the final fragments contract at a rate determined by the energy leak down their equilibrium temperature gradients.

A number of objections can be raised against Hoyle's scheme and his arguments for it. There is a logical hiatus between "can" and "will": the fact that a blob would be gravitationally bound even if it were not part of a larger mass does not prove that it will in fact separate out from the collapsing background. On the contrary, if the cloud remained, e.g., a strictly uniformly dense sphere, then no fragments would form. One needs to emphasise that there will inevitably be a field of pressure-density fluctuations of all wavelengths up to the cloud radius - "longitudinal turbulence". The problem is then to give dynamical reasons why fluctuations of large enough scale should be amplified more rapidly than the mean background density. Appeals to the necessity to dissipate energy are not convincing unless supported by a dynamical model. Further, although dissipation inevitably occurs, we have noted that in order to end up with a star cluster of finite radius, we require that dissipation be cut off by the very process of star formation; emphasis on energy dissipation is therefore misleading.

Consider, then, a density fluctuation within the contracting cloud. If the background motion could be ignored, the Jeans analysis yields as the vertical length for amplification

$$\lambda_J \simeq c \left(\frac{\pi}{4G\rho}\right)^{1/2} \propto R^{3/2}. \qquad (27)$$

Thus $\lambda_J / R \propto R^{1/2}$, so that the Jeans length becomes a smaller and smaller fraction of the radius as contraction ensues. This is just another way of stating Hoyle's point that the minimum mass allowed by the virial theorem decreases under isothermal contraction, and does not itself prove that such a blob will separate out from the contracting background. However, one's first guess is that a local density excess will cause a local gravitational pull towards the center of the blob, so that the local density goes up more rapidly than the mean. This argument has been challenged on time-scale grounds. The time of free-fall of the whole cloud from an initial density ρ_0 is $\simeq (G\bar{\rho}_0)^{-1/2}$. Within a blob the pressure gradient will be necessarily larger than in the whole cloud, and will therefore reduce gravity somewhat, but for wavelengths well above the instantaneous Jeans length this can presumably be ignored, so that the time of collapse of the blob due to its self-gravitation is $\simeq [G(\bar{\rho}_0 + \delta\rho_0)]^{-1/2}$, where $(\bar{\rho}_0 + \delta\rho_0)$ is the initial density of the blob. If $(\delta\rho_0/\bar{\rho}_0) \ll 1$ — so that small perturbation theory is applicable — the two times are almost identical; hence it is argued that fluctuations will not grow more rapidly than the mean background density.

However, this argument depends on misusing the idea of a "free-fall time" - the time in which the density of a collapsing blob increases by a substantial factor. The near equality of the two times, for the cloud and for the small blob within it, means not that the blob cannot separate out, but that one must wait a time of the order of the common free-fall time before the extra self-gravitation of the blob manifests itself. As an example, consider a collapsing uniform sphere, of density $\bar{\rho}_0$, and a sub-sphere within it, of density $(\bar{\rho}_0 + \delta\rho_0)$. The mean gravitational field yields the same relative inward acceleration for each element - in (15) $\bar{\rho}_0(m)$ is by hypothesis the same $(\bar{\rho}_0)$ for each mass sphere, so that the associated parameter $\bar{\theta}$, and therefore r/r_i, are functions of time only. Thus the background gravitational field does not disrupt the blob. As we are concerned with times near the end of the free-fall of cloud, we replace $\bar{\theta}$ by $\pi/2 - \bar{\chi}$ so that the mean density is by (15)

$$\bar{\rho} = \bar{\rho}_0 \operatorname{cosec}^6 \bar{\chi} \simeq \bar{\rho}_0 / \bar{\chi}^6, \tag{28}$$

and $\bar{\chi}$ at time t satisfies

$$t\left[\frac{8\pi}{3} G\bar{\rho}_0\right]^{1/2} = \frac{\pi}{2} - \bar{\chi} + \sin\bar{\chi}\cos\bar{\chi} \simeq \frac{\pi}{2} - \frac{2}{3}\bar{\chi}^3 \tag{29}$$

Introducing a similar parameter $\chi = \frac{\pi}{2} - \vartheta$ to describe the density of the blob as a function of time, we have

$$t\left[\frac{8\pi}{3}G(\bar{\rho}_0 + \delta\rho_0)\right]^{1/2} \simeq \frac{\pi}{2} - \frac{2}{3}\chi^3; \tag{30}$$

hence,

$$\left(\frac{\bar{\rho} + \delta\rho}{\bar{\rho}}\right)_t = \left(\frac{\bar{\rho} + \delta\rho}{\bar{\rho}}\right)_0 \Big/ \left[1 - \frac{3\pi}{8\bar{\chi}^3}\left(\frac{\delta\rho}{\bar{\rho}}\right)_0\right]^2, \tag{31}$$

so that even if $(\delta\rho/\bar{\rho})_0 \ll 1$ the local density has become large compared with the mean when

$$\bar{\chi} \simeq \left(\frac{\delta\rho}{\bar{\rho}}\right)_0^{1/3}, \tag{32}$$

i.e. when the relative contraction of the cloud

$$\frac{r(m,t)}{r(m,0)} = \cos^2\bar{\vartheta} \simeq \bar{\chi}^2 \simeq \left(\frac{\delta\rho_0}{\bar{\rho}_0}\right)^{2/3}. \tag{33}$$

For example, if $(\delta\rho/\bar{\rho})_0 \sim 10^{-3}$, $\frac{r}{r_0} = 10^{-2}$. The point is that the relevant independent variable is not the time, for gravitational collapse accelerates rapidly, but rather the degree of contraction of the whole cloud. The singularity in (31) when $\bar{\chi}^3 = \frac{3\pi}{8}\left(\frac{\delta\rho}{\bar{\rho}}\right)_0$ means that once the free-fall time has elapsed the fragments rapidly acquire negligibly small cross-sections. If, as is more plausible, the density fluctuations present are not small, then the separating out of blobs will occur earlier during the collapse.

Crucial steps in this argument are (I) the choice of wavelengths large compared with the instantaneous Jeans length, so that pressure is ignorable for the blob as well as the cloud; (II) the recognition that the mean field is not disruptive in a spherical system. This "tidal effect" of the mean field is the second argument that has been used against fragmentaion.[8] In a less idealized problem one would expect that even if the cloud as a whole is collapsing roughly spherically, the departures from strict spherical symmetry would probably mean that the mean gravitational field is able to prevent amplification of some small density fluctuations. If the local mean field changes its direction sharply across a blob of scale l, the disruptive force could be as large as $G\bar{\rho}l$; the blob would then be able to grow - against the mean background - only if $\bar{\rho} + \delta\rho \simeq 2\bar{\rho}$ - ie. only if the perturbation is "large". However, one does not need the fragmentation process to be 100 per cent efficient, and it is difficult to believe that in a cloud which is collapsing roughly spherically, the possible tidal disruption would be the dominant effect, especially as there is no physical reason for limiting the density perturbations to a small fraction of the mean.

A rigorous treatment of the instabilities in a freely-falling, uniform, isothermal sphere has been given by C. Hunter.[9] He considers a small arbitrary perturbation, which has both an irrotational and a solenoidal part. He confirms that the "tidal" term is not disruptive, and that the rough treatment given above does predict correctly the order of the mean density at the epoch when the blob has effectively separated out. If pressure is initially important - ie the perturbation considered has a scale less than the initial Jeans length - then at first the density oscillates; but as the mean density goes up secular amplification takes over. The separating out is delayed but not prevented: perturbations which are initially "stable" ultimately become unstable. Further, amplification occurs only when the self-gravitation of a blob is large compared with the blob pressure gradient, in agreement with our rough treatment, but contrary to Hoyle's argument that fragmentation allows the cloud to dissipate significantly more energy.

Hunter also considers the non-isothermal problem. If $1 < \gamma < 4/3$, the pressure term is asymptotically negligible, so that the pressure-free solution is ultimately valid. If $5/3 > \gamma > 4/3$ the solutions are oscillatory, with amplitudes and frequencies that increase with time. If $\gamma = 4/3$ perturbations with wavelengths initially less than the Jeans length also yield oscillatory solutions.

It should be noted that the epoch - measured by the appropriate parameter, the radius of the cloud - at which a perturbation has amplified significantly depends on the perturbation strength. Thus although the length scale must exceed the instantaneous Jeans length for secular amplification, it does not follow that Hoyle's hierarchy is the correct picture. Rather the mass spectrum of the self-gravitating blobs that form depends on the spectrum of the longitudinal turbulence present in the cloud. If the turbulent kinetic energy is concentrated in the smaller wavelengths, then their more rapid growth, once the Jeans length is small enough, may more than off-set the earlier start of the larger wavelengths. Any local cooling will further assist collapse of small blobs. Observationally, the hierarchical picture seems to apply more to a cluster of galaxies, each with its stellar sub-systems, than to a star cluster.

If the perturbation field had too high a degree of symmetry, all the blobs would ultimately coalesce near the center, and nothing would be gained. However, even if the cloud as a whole has zero angular momentum, the perturbations in the velocity field will in general endow each fragment with some angular momentum about the cloud's mass-center; and as the fragments rapidly acquire very small cross-sections, the kinetic energy that keeps the cluster of finite extent is not destroyed by collisions. The only doubt that arises is whether the random gravitational fields associated with the density fluctuations would endow each fragment with so much spin that its contraction to a body of small cross-section would be seriously impeded, thus increasing the probability of energy destruction by collision.[10] However, the neglect of the angular momentum in the cloud is highly unrealistic, so it is anyway unplausible to extrapolate the theory to indefinitely high mean densities. The value of Hunter's work - apart from its clarifying our ideas on a classical problem - is twofold. First, it may very well apply to the initial stages of the contraction of a cloud of finite angular momentum, before the problem is fundamentally changed by centrifugal force becoming comparable with gravity: e.g. it may describe the break-up of the primeval meta-galactic cloud into proto-galaxies. Secondly, when dealing with further complications, such as magnetism, it gives one confidence to bridge the gap between "can" and "will"; one expects that density fluctuations of a scale substantially larger than the minimum set by the appropriate form of the virial theorem will always be able to grow against a contracting background.

1.5 Accretion by previously formed stars.

Gravitational accretion differs from gravitational instability, in that it deals with the effect on gas, presumed to have negligible self-gravitation, of the gravitational field of a star already formed. The process is most efficient when the star is moving through the cloud at a speed much less than the sound speed c (~ 1 km/sec. at 10^2 °K). Then if again magnetism and angular momentum are assumed innocuous, the gas flows approximately spherically towards the star, and the rate of increase of the mass M of the central star is[11]

$$\frac{dM}{dt} = 4\pi\lambda\left(\frac{GM}{c^2}\right)^2(\rho_\infty c) = 4\pi\lambda r_B^2\rho_\infty c, \qquad (34)$$

where

$$r_B = \text{Bondi radius} = \frac{GM}{c^2};$$

ρ_∞ = the density far from the star, and λ is a parameter of order unity, its exact value depending on the effective value of γ for the flow. For isothermal flow this formula can be derived approximately as follows. Near enough to the star, the gas flows under the star's gravitation unimpeded by pressure, so that its speed v at radius r is given by

$$v^2 \simeq \frac{2GM}{r} . \tag{35}$$

In a steady state

$$4\pi r^2 \rho v = A = \text{constant accretion rate}, \tag{36}$$

so that the pressure gradient per unit mass is

$$-\frac{c^2 \rho'}{\rho} = c^2\left(\frac{v'}{v} + \frac{2}{r}\right) \simeq \frac{3}{2}\frac{c^2}{r} \tag{37}$$

by (36). Thus the pressure term ceases to be small compared with gravity when

$$\frac{3}{2}\frac{c^2}{r} \simeq \frac{GM}{r^2}, \tag{38}$$

or $r \simeq 2r_B/3$, at which radius, by (35), $v \simeq c$. Beyond $r \simeq r_B$ the inertia of the flow is negligible, $\rho \simeq \rho_\infty$, and $v \simeq c\,(r_B/r)^2$; within r_B, $\rho \simeq \rho_\infty (r_B/r)^{3/2}$. The accretion rate $A = 4\pi\rho v r^2 \simeq 4\pi \rho_\infty c\, r_B^2$, in order of magnitude agreement with Bondi's accurate formula (34).

If $\Theta \simeq 10^2\ {}^\circ K$ and $\rho \simeq 10^3\ m_H / cm^3$, then (34) shows that the accretion rate is sufficient to build up a massive star in a reasonable time. But a bright star moving into a cool $H\,\underline{I}$ cloud tends to build up a hot zone of ionized hydrogen.

Even if the hot sphere achieves dynamical equilibrium with the surrounding cooler matter, so that inflow starts up, the pressure of the gas, now at $10^4\ {}^\circ K$, will cut down the accretion rate by a factor of at least 10^3 - more if the density has been reduced by the establishment of equilibrium. The only way for accretion to occur at the maximum rate is for the Strömgren sphere to be kept small - somewhat less than the Bondi radius computed for c the sound speed at $\underline{10^4\ {}^\circ K}$. This will occur if the density near the star is much higher than the density far away - so that the extinction of ultraviolet light is greatly increased.[12] This can be achieved if the star is initially of low luminosity, so that it can build up its accretion density-velocity field. A small Strömgren sphere is built up; it gradually expands as the ultra-violet flux increases with the mass of the star, until at a critical mass - depending on the mean density of the cloud - the H II zone explodes, and the cloud is effectively exposed to a newly-born O or B star.

This process for producing O and B stars effectively separates the problem of the origin of these relatively few bright (and therefore short-lived) stars, from the origin of the far more numerous stars of moderate mass. Such a separation was more plausible ten years ago, before we had convincing evidence - e.g. from H-R diagrams of young clusters - that the whole luminosity function is being formed "today", long after the condensation of the bulk of the galactic gas $5\text{-}10 \times 10^9$ years ago. However, there is one theoretical reason for taking note of the accretion mechanism. A good deal of current work on the dynamical consequences of the heating of interstellar gas by O and B stars does apparently demand that the stars be suddenly "switched on". It is, however, not clear that a contracting proto - O or B star approaches the main sequence quickly enough for its ultra-violet flux to start in a time comparable with the dynamical time-scales demanded - e.g. with the 10^4 years that Kahn and Menon[13] find for the age of the Orion nebula; whereas on the accretion model, the ultra-violet flux is available as soon as the small Strömgren sphere is forced to expand.

However, the estimate of the accretion rate we have given is the best possible - we have ignored factors such as magnetism and angular momentum that certainly cause difficulty. Even so, as pointed out by Kahn,[14] we cannot ignore the dynamics of the cloud in which the star is embedded. Bondi's formula (34) yields the time

$$\frac{c^3}{2\pi\lambda G^2 \rho_\infty M} \qquad (39)$$

for the star to double its mass. Now suppose the cloud is gravitationally bound, but still fairly transparent, so that it is collapsing in a time $\simeq (6\pi G\rho_\infty)^{-1/2}$. Then the accretion time-scale is less than that of gravitational collapse only if

$$M \ll \frac{4\pi}{3}(2\lambda^2)\left(\frac{GM}{c^2}\right)^3 \rho_\infty \qquad (40)$$

i.e. if M is less ~~than~~ the mass in Bondi's sphere; in which case we are dealing with collapse of the cloud unders its own gravitation - the mass of the star is merely a small perturbation.

On the other hand, if the cloud is not gravitationally bound, but is expanding at the speed of sound under its own pressure, then

$$\frac{4\pi}{3} G \rho_\infty R^2 < c^2 , \tag{41}$$

R being the cloud radius. The condition that the accretion time be less than the expansion time R/c reduces to

$$\frac{GM}{c^2} > R_c \tag{42}$$

approximately - i.e. the Bondi sphere embraces the whole cloud, which therefore cannot contain more than a fraction of the stellar mass. Only if the cloud is prevented from disrupting by an external pressure could accretion be significant.

There is one further possibility. Suppose the cloud is gravitationally bound and has collapsed far enough for some small proto-stars to have formed, but it is now opaque enough to be kept in approximate dynamical equilibrium by thermal pressure. Then although further sub-condensation is prevented by pressure, the proto-stars may increase their mass by accretion; as shown by Bondi, gas can flow into the local potential well due to the star's gravitation field, even if the opacity is high enough to keep the flow adiabatic. With $\gamma = 5/3$, the parameter λ in Bondi's formula (34) is $1/4$. If the cloud of mass m and radius R is in equilibrium,

$$c^2 \simeq \frac{Gm}{R} , \tag{43}$$

and

$$\rho_\infty \simeq \frac{m}{\frac{4\pi}{3} R^3} ; \tag{44}$$

the accretion rate is then

$$\frac{4\pi (GM)^2}{4\left(\frac{Gm}{R}\right)^{3/2}} \frac{m}{\frac{4\pi}{3} R^3} = \frac{4\pi (GM)^2}{\frac{16\pi}{3} (GR)^{3/2} m^{1/2}} \propto \frac{1}{R^{3/2}} ; \tag{45}$$

the increase in the cloud density more than off-setting the increased sound speed. For example, if the cloud of mass $10^3 \odot$ is kept in equilibrium by a temperature of $10^3\,°K$ - at a radius of about 1/2 parsec - the maximum accretion rate is two orders of magnitude above the rate in a cloud at $10^2\,°K$ and a density of $10^3 m_H/cm^3$. It is therefore possible that the formation of the massive stars in a cluster may occur in two stages -gravitational instability followed by accretion. An expanding association can arise if only a fraction of the cloud is able to condense into stars. If enough O or B stars are formed - either directly or by accretion - to heat the uncondensed gas up to $10^4\,°K$ and so blow it away, then the mass condensed into stars may not be able to stay bound, so that the whole association expands.

2. Angular Momentum and Magnetism - General Considerations

In Chapter 1 we discussed the circumstances that are most favourable for star formation. The problem was comparatively simple, in that the forces acting - pressure and gravity - are essentially isotropic. Even if the perturbations have a rotational component, Hunter's analysis[4] shows that the initial growth of the rotational part — due to conservation of circulation under the mean flow — is slower than the growth of the irrotational part — due to the self-gravitation of the blob.

We now discuss in general terms the consequences of intrinsically non-isotropic properties, angular momentum and a magnetic field. Here each is studied separately; in Chapter 3 we consider their joint effect and their mutual interaction.

2.1. Rotation - its overall effect.

The most striking consequence of the centrifugal field in a cloud that conserves its angular momentum is its limitation of isotropic contraction. For consider a gravitationally contracting sphere of mass M, radius R, and an angular velocity Ω that must satisfy

$$\Omega R^2 = \text{constant} . \tag{1}$$

Then in spherical polar coordinates (r, θ, ϕ) based on the center and the rotation axis, the ratio of the centrifugal force to the opposing component of gravity at (R, θ, ϕ) is

$$\frac{\Omega^2 (R \sin\theta)}{(GM/R^2)\sin\theta} = \left(\frac{\Omega^2 R^4}{GM}\right)\frac{1}{R} , \tag{2}$$

which inevitably approaches unity as R decreases. If the sphere is non-uniform in density but rotating uniformly, centrifugal balance for each sphere $m(r)$ is reached when

$$1 = \frac{\Omega^2 r^3}{G m(r)} = \frac{\Omega^2}{\frac{4\pi}{3} G \bar{\rho}} , \tag{3}$$

where again $\bar{\rho}(r)$ is the mean density within r.

As an example consider a cloud taking part in the local rotation of the galaxy $(\sim 10^{-15}/\text{sec})$. Then if $\rho \simeq 10^3 m_H \simeq 10^{-21}$, $\Omega^2 / \frac{4\pi}{3} G\rho \simeq 2 \times 10^{-3}$, so that if such a cloud is gravitationally bound, its spherically symmetric contraction will be halted after its radius has shrunk by about 1/500.

The customary formulation of this result is as follows. The centrifugal parameter

$$\eta = \frac{\Omega^2}{\frac{4\pi}{3} G \rho} \propto \frac{1}{R} \propto \rho^{1/3}; \tag{4}$$

hence in contracting from $\rho \simeq 10^{-21}$ to $\rho \simeq 1$ (a rough main sequence value), η for a sphere increases by 10^7. If the sphere were to reach the main sequence, the rotational speed at its equator would increase by the same factor. The radius of a blob of mass $\odot$ at a density $5/3 \times 10^{-24} \simeq 7 \times 10^{17}$ cms, so that the initial rotational speed is $\simeq 7 \times 10^2$ cm/sec; hence at the main sequence the blob would have an equatorial speed at the surface approaching that of light. The reductio ad absurdum, however, derives not from the words "speed of light," but from the enormous centrifugal forces that, as already seen, would not allow spherical contraction beyond a radius far above a main sequence value.

As the component of gravity parallel to the axis of rotation is not reduced, the sphere must begin to flatten, with the gravitational energy released becoming initially kinetic energy of z-motion. The problem of the subsequent break-up of such a cloud is postponed; here we note that the asymptotic structure of the rotating cloud again depends strongly on how much of the gravitational energy released is dissipated (and radiated away), and how much is retained as the kinetic energy of sub-condensations.

If the z-energy that is undissipated is small compared with the rotatory energy, the cloud will approximate to a flattened spheroid or a disk. The most obvious application of this case is to the disk-like galaxies. To study their overall structure, it is convenient to idealise them as infinitely thin and axially symmetric. The simplest case is that with the mass per unit area given by

$$M = M_0 \left[1 - \left(\frac{\varpi}{R}\right)^2 \right]^{1/2}, \tag{5}$$

and where R is the radius and ϖ radial distance, M_0 constant. It is well know[15] that such a disk is maintained in centrifugal equilibrium by a uniform rotation Ω_0 where

$$\Omega_0^2 = \frac{\pi^2 G M_0}{2R}. \tag{6}$$

It is clear that the disk (5) arises if a uniformly rotating sphere of uniform density is flattened, and distances in the plane are all changed by the same factor, with each element of matter conserving its angular momentum.

However, in addition there exists another equilibrium state, derivable from (5) and (6) by a non-uniform contraction, in which again each element conserves its angular momentum.[16] In the inner regions the transformation is

$$\varpi' \simeq \frac{\pi}{2}\frac{\varpi^2}{R}, \tag{7}$$

yielding the approximate area-density law

$$M'\varpi' = \frac{M_0 R}{\pi} = \text{constant}, \tag{8}$$

and rotation law

$$\Omega'\varpi' = \left(2\pi G M'\varpi'\right)^{1/2} = \left(2GM_0R\right)^{1/2} = \text{constant}. \tag{9}$$

Whereas in (5) the gravitational field depends critically on the mass outside the radius considered, the field in (8) is approximately Keplerian: i.e., derivable by ignoring the mass outside ϖ', and concentrating all the mass within ϖ' at the center. It is remarkable that these two equilibrium states, one with solid body rotation and one with a nearly uniform rotational velocity, can both be "derived" from the primeval uniformly rotating, uniform sphere, under the constraint of strict detailed conservation of angular momentum. Each disk appears to be stable against small perturbations tending to transform it into the other disk. Which state the primeval cloud approaches presumably depends on the large-scale perturbations in the initial near-spherical state. The results are of particular interest because of recent observations by the Burbidges and Prendergast.[17] They do in fact find that most of the disk-like galaxies divide sharply into two classes, one with approximate solid body rotation, and the other with approximate uniform rotational speed over a wide range (e.g. our own galaxy and Andromeda).

However, besides the disk-like galaxies there are the ellipticals; these also presumably have a net angular momentum, but the much greater mean z-dispersion of their stars implies that more of the gas condensed into stars before much z-energy dissipation occured.

Within our own galaxy, the only highly flattened system we know is the solar system: star clusters may show some ellipticity, but certainly not a disk-like structure. Thus at the same time as we attempt to build stars within a rotating cloud, we have to explain the different ultimate structure of systems all endowed with a dynamically significant angular momentum.

2.2. Sub-condensation in a rotating cloud.

Consider again the uniformly rotating sphere flattening parallel to the axis, so that its spin Ω stays constant. Imagine a spherical blob within it, of radius r, separating out from the background. (Again we ask first the question "Can it condense?", and later, "Will it?") Then if the background density has increased to ρ the ratio of the centrifugal force of spin to the opposing component of the blob's gravity is

$$\frac{\Omega^2 r}{G\left(\frac{4\pi}{3}\rho r^3\right)/r^2} = \frac{\Omega^2}{\frac{4\pi}{3}G\rho} \tag{10}$$

i.e. it is again the parameter η of (4). Thus, as expected, when the overall centrifugal force is able to halt isotropic contraction of the whole cloud – $\eta = 1$ – so it also prevents a sub-sphere from separating out. But once significant flattening has occured $(\eta \ll 1)$, then a sub-sphere massive enough to overcome its thermal pressure can begin to contract isotropically. Thus suppose the sphere has collapsed uniformly parallel to the axis, so that a cylindrical section of the sphere originally of length $2L$, radius r, density ρ_0 and mass M has become effectively a sphere of radius r and of density $\bar{\rho}$.

Since

$$M = 2\pi\rho_0 L r^2 = \frac{4\pi}{3}\bar{\rho} r^3, \tag{11}$$

then

$$\bar{\rho} r \simeq \rho_0 L \tag{12}$$

(factors of order unity are dropped in this rough treatment). If ρ_0 is the density at which centrifugal balance holds — $\eta = 1$ — we have

$$\eta = \frac{\Omega^2}{\frac{4\pi}{3}G\bar{\rho}} = \frac{\Omega^2}{\frac{4\pi}{3}G\rho_0}\left(\frac{\rho_0}{\bar{\rho}}\right) \simeq \frac{r}{L} \tag{13}$$

Thus the greater the flattening the smaller is the centrifugal parameter η so that the sphere r can contract correspondingly further before in its turn it feels the effect of the increasing centrifugal force of spin. But this softening of the effect of rotation is achieved - in this "cylindrical" geometry - at the cost of a corresponding reduction in the mass of the sphere. For example, suppose we require that the sphere is to contract to a main sequence radius r_m (for which $\rho_m \simeq 1$), still conserving its angular momentum, but without centrifugal force becoming too large. Then

$$\left(\frac{r}{r_m}\right)^3 \simeq \frac{\rho_m}{\rho} \simeq \left(\frac{r}{L}\right)\left(\frac{\rho_m}{\rho_0}\right), \tag{14}$$

so that

$$1 > \frac{\Omega_m^2 r_m^3}{GM} = \left(\frac{\Omega^2 r^4}{GM}\right)\frac{1}{r_m} = \left(\frac{\Omega^2}{\frac{4\pi}{3}G\rho}\right)\left(\frac{r}{r_m}\right) \simeq \left(\frac{r^2}{r_m L}\right) \tag{15}$$

or

$$1 > \left(\frac{r}{L}\right)^{4/3}\left(\frac{\rho_m}{\rho_0}\right)^{1/3}, \tag{16}$$

where (13) and (14) have been used. The mass of the sphere is

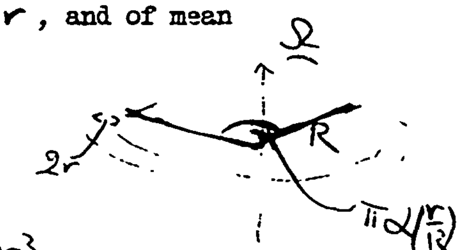
$$2\pi\rho_0 L^3\left(\frac{r}{L}\right)^2 < 2\pi\rho_0 L^3\left(\frac{\rho_0}{\rho_m}\right)^{1/2}. \tag{17}$$

Thus the larger we require ρ_m to be - or the greater the degree of contraction - then the smaller is the allowed mass of the sphere.

As an extreme case, imagine all the mass of the galaxy spread over a uniformly rotating primeval sphere, of a radius such that with the prescribed angular momentum, the sphere is in centrifugal balance ; ρ_0 is then $\simeq 10^{-24}$, and the radius $\simeq 3\times 10^{22} = 10$ kpc. Then if (15) is to hold (with $\rho_m \simeq 1$), $(r/L) < 10^{-6}$, so that $r < 10^{-2}$ parsecs (taking 10 kpc for L — an upper limit). Then by (16), $M < \odot/10$. Thus even with the most extreme assumptions it is impossible to build in this way any but very faint dwarf stars. The situation is no better for present-day gas clouds. Any body formed in this way, and of mass substantially above $\odot/10$, would itself flatten to a disk before reaching the main sequence. If the body has become opaque enough to build up a strong thermal field, then it will probably become rotationally unstable and eject matter from the surface. Ignoring this, one can imagine a new hierarchy forming, in which the smallest blobs perform a sort of epicyclic motion: they orbit around the mass — center of a disk-like structure, which itself orbits around the center of a larger disk- etc.

One is tempted to conclude that these results are again a reductio ad absurdum of the assumptions. However, although we cannot build stars even of moderate mass in this way, and so must explore different models, it does not follow that such cylindrical contraction, followed by spherical collapse, does not occur. We cannot assert that the gas in a gravitationally bound cloud must either condense into stars (getting over, somehow, the angular momentum difficulties) or remain uncondensed. It is not inconceivable that when the disk population formed, a substantial fraction of the gas broke up into masses of planetary rather than stellar order. Such bodies would be observable only through their integrated gravitational effect. The work of Oort and his collaborators has shown that the estimated stellar and gaseous components in the solar neighbourhood do not exert enough gravitational force to explain the observed distribution of K-giants above the galactic plane - an extra 30 per cent is needed. Unless the proportion of molecular hydrogen (undetectable by 21 cm. measurements) is much greater than the atomic, then it does seem that, at least near the sun, the luminosity function should be extended to much lower masses. And although we shall try to see how stars can form in spite of the angular momentum difficulty, one cannot be sure that the suggested processes always operate, so that we can ignore altogether the straightforward cylindrical collapse that leads to the ultimate formation of small masses.

Suppose now that after our rotating sphere has flattened, a spherical blob is formed by the agglomeration of the mass in a section of a torus of radius r, and of mean distance R from the axis of rotation. If the length of the segment is $\pi\alpha r$ where $2 < \alpha < 2R/r$ the mass of the sphere so formed is



$$M = (2Lr)\rho_0(\pi\alpha r) = 2\pi\rho_0 L r^2 \alpha = \frac{4\pi}{3}\bar{\rho} r^3 . \qquad (18)$$

Again ignoring any shear, we have that the sphere will orbit with the local angular velocity Ω, and also spin at the rate Ω before it begins to contract. The advantage of this geometry is that much larger masses can form than by simple cylindrical collapse, without running into centrifugal trouble during the approach to the main sequence. For at the main sequence we have, instead of (15),

$$1 > \frac{\Omega_m^2 r_m^3}{GM} \simeq \left(\frac{1^2}{r_m L}\right)\frac{1}{\alpha} . \qquad (19)$$

Again with $\rho_m \simeq 1$,

$$\left(\frac{r}{r_m}\right)^3 \simeq \left(\frac{r}{\rho_0 L \alpha}\right), \qquad (20)$$

so that (16) is replaced by

$$r < \alpha \left(L \rho_0^{1/4}\right) \qquad (21)$$

approximately, and

$$M < \alpha^3 (M)_{\alpha=1} . \qquad (22)$$

Since in principle α could be as large as $2R/r \simeq 2L/r$, there could appear a large factor $(L/r)^3$ in (22). Thus masses of stellar order could condense even with a substantial reduction in L —we need not require that the whole height of the section of a cylindrical shell becomes part of just one spherical mass.

2.3. Dynamical problems in a rotating, non-magnetic cloud.

So far we have merely looked at different geometries of condensation to see which could yield a mass with low enough spin for substantial contraction to be possible. We now return to the systematically flattening, uniformly rotating sphere, and discuss qualitatively its possible instabilities.

As in the problem without rotation, one can treat simply the idealized case of an infinite uniform medium of density ρ, with the additional condition that the whole medium rotates with uniform angular velocity Ω . Chandrasekhar[16] solved the "Jeans" problem for this case, using a frame of reference rotating with the medium. He showed that except for waves travelling perpendicular to the axis, the Coriolis force does not affect the Jeans criterion. In the exceptional case, the waves are stabilized if $\Omega^2/\pi G\rho > 1$; if $\Omega^2/\pi G\rho \ll 1$, the Jeans criterion is adequate. However, again one can consider these results only as suggestive, confirming our earlier conclusions as to when rotation does not positively impede initial local density growth; but as soon as we depart from the physically dubious infinite medium, we are forced to admit that the cloud is itself collapsing parallel to the axis.

The problem is in one respect simpler than the spherical case; for given a finite pressure p (either thermal, turbulent or magnetic), the collapse towards an infinitely thin disk is halted at a finite thickness. If $p = c^2 \rho$, the equilibrium semi-thickness is approximately[19,20]

$$\bar{z} = \left[\frac{c^2}{2\pi G \rho} \right]^{1/2} \tag{23}$$

— just the order of the Jeans length. Hence provided the cloud dissipates enough z-energy while staying uncondensed, we can define a steady state - the rotating disk - to which ordinary stability analysis can be applied. We begin by discussing this case.

Chandrasekhar's analysis is not valid here, since the scale of variation in the z-direction, so far from being large compared with the wavelengths considered, is of the same order as the Jeans length. However, one can see at once that in the flattened state Coriolis force is small compared with the perturbations in gravity. A density perturbation $\delta\rho$ is related to a perturbation velocity v by the equation of continuity;

$$\frac{\rho v}{\bar{z}} \simeq \frac{\delta\rho}{T} . \tag{24}$$

Here T is the time of the instability, which is at least $(G\rho)^{-1/2}$; the scale of variation of the unstable modes is at least $\simeq \bar{z}$ (the Jeans length). Thus the desired ratio is

$$\frac{v\Omega}{G\delta\rho\bar{z}} \simeq \frac{\Omega}{G\rho T} \simeq \left(\frac{\Omega^2}{G\rho}\right)^{1/2} \tag{25}$$

In the original spherical state this ratio is of order unity; subsequent collapse parallel to the rotation axis increases ρ without changing Ω. In fact for the disk approximation to be valid, $\bar{z}$ is much less than the cloud radius R: hence

$$c^2 \simeq 2\pi G\rho\bar{z}^2 \simeq 2\pi G\rho_0 R\bar{z} \ll 2\pi G\rho_0 R^2 \simeq \frac{GM}{R}, \tag{26}$$

where M is the cloud mass. Thus provided the spherical cloud is well past the stage at which thermal pressure is important before it begins to flatten, we can be sure that the ratio (25) is small.

Ledoux[20] has treated the gravitational instability of such a flattened system, ignoring Coriolis force; he confirms that wavelengths equal to a slightly modified Jeans length are marginally unstable. The break-up of a disk into rings of scale rather larger than the thickness has also been discussed.[21] What does seem lacking is a treatment of azimuth-dependent perturbations, $\propto e^{im\phi}$ (m integral), of scale ℓ greater than the Jeans length. The following rough treatment suggests that the e-folding time increases with ℓ, at least for torus sections of radius $\simeq \bar{z}$. The gravitational acceleration along such a torus section is approximately

$$\frac{G(\rho \bar{z}^2 \ell)}{\ell^2} = \frac{G\rho \bar{z}^2}{\ell} \simeq \frac{\ell}{\tau^2}, \tag{27}$$

yielding

$$\tau \simeq \left(\frac{\ell}{\bar{z}}\right)(G\rho)^{-1/2}. \tag{28}$$

Thus provided ℓ is sufficiently above $\bar{z}$ for the pressure to be negligible, the collapse time increases with ℓ: there should be a length of the order of $\bar{z}$, though somewhat larger, that yields minimum e-folding time. It is therefore unlikely that the torus condensation, discussed in 2.2, will occur simply by gravitational instability: rather one expects approximately spherical blobs to separate out and begin to contract. Subsequent collisions between such blobs, due to perturbations in their orbital velocities, could conceivably lead to agglomeration and the build-up of larger masses without much spin angular momentum. But if the disk is very thin - low temperature - so that $\Omega^2/\pi G\rho \ll 1$ - the spherical blobs can acquire a very small cross-section before their spin interferes with their collapse, so that the chance of mutual collisions is much reduced.

It should be noted that non-uniform rotation introduces considerable mathematical difficulties, as it is impossible to remove the zero-order motion from the problem by choice of a rotating frame. Physically, the difficulty is again due to the effect of the mean gravitational field on the perturbation - it tends to enforce the zero-order shear, and so off-set the effect of the blobs' self-gravitation. However, it is not difficult to imagine types of motion that will nullify the shear. Thus consider two circles, distant ϖ_1 and ϖ_2 from the center, with angular velocities Ω_1 and Ω_2, displaced to new radii

$\tilde{\omega}_1'$ and $\tilde{\omega}_2'$; then under conservation of angular momentum, the new angular velocities Ω_1' and Ω_2' are equal if

$$\left(\frac{\tilde{\omega}_2'}{\tilde{\omega}_1'}\right)^2 = \left(\frac{\Omega_2}{\Omega_1}\right)\left(\frac{\tilde{\omega}_2}{\tilde{\omega}_1}\right)^2. \tag{29}$$

If $\tilde{\omega}_2 > \tilde{\omega}_1$, $\Omega_2\tilde{\omega}_2^{\,2}$ must exceed $\Omega_1\tilde{\omega}_1^2$ for the unperturbed disk to be stable against turbulence (Rayleigh's criterion). Hence $\tilde{\omega}_2' > \tilde{\omega}_1'$ – i.e. one can achieve a uniformly rotating ring without forcing the particles to pass through each other. If $\Omega_2 < \Omega_1$ (the usual case) the circles move towards each other, increasing the density. In the uniformly rotating ring so formed, one expects instabilities similar to those in the uniformly rotating disk.

We now return to the more difficult problem of the instabilities within the rotating cloud as it collapses towards the disk-like state. This is the analogue of Hunter's problem for the non-rotating cloud: the problem of the growth of perturbations against the background density that is itself growing at the free-fall rate. As yet no rigorous treatment of this problem has appeared; but by analogy with Hunter's results, one can argue as follows. Waves with components parallel to the axis are in any case unimpeded by rotation; one expects that right from the start such perturbations of scale greater than the Jeans length will amplify steadily against the background; though again if the density fluctuations are small, the cloud as a whole will have approached a disk before a really striking corrugation in the z-direction is achieved. Further, once the centrifugal parameter $\Omega^2/\pi G\rho$ has decreased to well below unity, waves of length greater than λ_J propagating across the axis will also amplify. In fact, once $\Omega^2/\pi G\rho \ll 1$, the conditions for a spherical blob to amplify against the background are in one way more favourable than in the problem without rotation; for the gravitational acceleration in the z-direction of the whole cloud is proportional to ρz, which is constant, whereas in spherical collapse this acceleration increases. As against this, we have that even if $\rho = 10\rho_0$ - a flattening by a factor 10 - a spherical blob cannot contract isotropically by more than about 1/3 before in its turn it is compelled to flatten through increasing spin. Though quantitative estimates are difficult, it seems probable that collisions between such blobs would destroy much of the z-energy released in the original collapse. Perhaps also collisions between blobs orbiting in the same ring may lead to agglomeration into a larger mass with essentially the same spin. All in all, one feels that a fairly uniform sphere collapsing parallel to the axis will tend to form ultimately a disk-like system, with small z-motions. If this treatment ignoring magnetic forces altogether is relevant, then for objects of high z-energy to separate out - e.g. the globular clusters in a disk-like galaxy - it appears that the initial sphere should have at the start strong density fluctuations, so that one does not have to wait for the system as a whole to flatten before any sub-condensation can form. In this way it may be possible for blobs of small enough cross section to form, so that the dissipation of energy in collisions is much reduced.

However, except for the possibility of torus condensation, which qualitatively does not seem very plausible, we are still without a process which gets over the "angular momentum problem," and allows stars to condense to the main sequence. We now must emphasize that if we relax the very severe constraint that a contracting sphere conserves all of its angular momentum, the problem is much simplified. For example, suppose that some mechanism keeps the spin of a contracting globule at about 10^{-15} sec^{-1} until it reaches the radius r', after which it conserves its angular momentum until the main sequence is reached. If the globule has a mass $\simeq \odot$ then r' need be no lower than 5×10^{16} cm $(\rho \sim 10^{-18})$ for the centrifugal force at the main sequence still to be less than gravity. Alternatively, suppose the sphere, formed as in 2.1 by collapse of the cylinder of height $2L$, contracts only to, say, 100 times its main sequence radius, after which angular momentum is systematically removed, and rapidly enough to keep the star stable. Then r/L must be less than $10^{-6} \times 10^{-1}$; and with $L = 3 \times 10^{22}$ the upper limit on the mass is increased to $\simeq 10^{2} \odot$. These figures illustrate how in principle one can get over the problem as to how a star can form at all; the more subtle question of the sharp drop at type F in main sequence rotation is for the moment ignored.

A purely hydrodynamic mechanism, such as that proposed by von Weizsäcker, by which turbulent friction transfers angular momentum to a dissipating envelope, runs into quantitative difficulties (quite apart from the problem of maintaining turbulence in an envelope with a Keplerian rotation law).[12] A more promising mechanism is magnetic braking: field lines emanating from a contracting blob into the surrounding medium tend to equalize the angular velocities of the blob and the medium. However, before studying this, one must first consider the overall effects of a magnetic field. And even if the magnetic field should turn out always to be an essential part of any actual process, it is necessary, if only for comparison, to try and get clear the dynamics of the evolution of the non-magnetic, rotating cloud.

It may be argued that the discussion we have given of the problem is adequate for a flattened disk, in which any turbulent velocities must be small compared with the circulatory, or to a collapsing cloud with only a weak turbulent field. If the turbulent pressure is strong enough to hold the cloud up temporarily, then the turbulent velocities must be comparable with the circulatory. With a strong vorticity field superimposed on the large-scale rotation, a blob of matter could find itself by chance with an abnormally low spin; if the blob is also massive enough it can collapse and form ultimately a main-sequence star, without running into centrifugal trouble on the way.

A difficulty here is that if the cloud is cool enough for small blobs to be able to collapse gravitationally, and yet with turbulent velocities comparable with the circulatory (and hence also the free-fall) velocity, then the turbulence must be highly supersonic. One's first guess is that its energy would be rapidly dissipated in shocks and lost, unless there is a source of energy - e.g. a hot star - that continually stirs up the gas. A different view has been urged by McCrea,[23] who pictures the cloud as a collection of "floccules" - small unbound blobs in random supersonic motion. A gravitationally bound mass within the cloud forms ultimately by collisions between those floccules which happen to be moving through the same small volume of space, so that the body built in this way necessarily has low spin, but high orbital angular momentum about the mass-center of the cloud. The basic kinematic idea, that a spherical star need not form from matter which was part of a sphere in the original cloud, is similar to the cylindrical and torus condensations already discussed.

McCrea's model is of great interest, and one hopes for further studies of its gas dynamics. It is certainly true that the rotating, strongly-flattened disk is the wrong model for the origin of star clusters, which through they undoubtedly have angular momentum, also have strong z-motion within the cluster (not to be confused with the z-motion of the whole cluster relative to the galactic plane). Presumably the ultimate test of any theory of star formation must be the luminosity function it predicts. Just as for incompressible turbulence we make statistical hypotheses whose consequences are tested experimentally, so it could be that by working backwards we could get some ideas on the properties of the supersonic turbulence that "decayed" into a star cluster. However, with the galactic magnetic field well attested, one feels that one must consider its role in the problem,especially in view of the well-known magnetic interference with eddying motion, and its interaction with large-scale rotation.

2.4 Gravitational collapse and fragmentation of a magnetic cloud.

Consider now a non-rotating cloud with a strong large-scale magnetic field $\underset{\sim}{H}$, "frozen in": the possible uncoupling of the field and matter will be taken up later. The overall dynamical effect of the field is contained in the Chandrasekhar-Fermi[24] extension of the virial theorem (cf. (1.8))

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2\,(\text{kinetic energy}) + \Omega + \mathfrak{M} + \int_S x_i\left(T_{ik} + p\,\delta_{ik}\right) dS_k, \qquad (30)$$

where T_{ik} is the magnetic part of the Maxwell stress tensor, and $\mathfrak{M}$ is the total magnetic energy within the volume bounded by S. Thus the internal magnetic energy has an essentially disruptive effect — the isotropic magnetic pressure $H^2/8\pi$ more than compensating for the unidirectional tension $H^2/4\pi$ along the field lines. If the field outside the cloud is strong, then the surface stresses T_{ik} help the thermal pressure p to stop the disruption, or accelerate the collapse of the cloud — 'magnetic pinching". However, once the cloud has contracted out of the background, conserving its magnetic flux, then we can again ignore all the surface terms. We shall therefore discuss the effect of the field on gravitational collapse by comparing the magnetic energy $\mathfrak{M}$ with the gravitational Ω.

As an example, consider a spherical cloud of mass M and radius R, with $H \simeq 10^{-6}$ gauss associated with a density $\rho \simeq 10\, m_H = 5/3 \times 10^{-23}$ gm/cm^3 — a rather lower estimate than some workers favour for the galactic field. Then gravitation can overcome magnetic resistance if

$$-\Omega = \frac{3}{5}\frac{GM^2}{R} > \left(\frac{4\pi}{3}R^3\right)\left(\frac{H^2}{8\pi}\right) = \frac{1}{6}H^2R^3, \qquad (31)$$

the factor $3/5$ corresponding to a uniform density. With $M = 4\pi\rho R^3/3$, we find as a lower limit to the mass that can be gravitationally bound:

$$M_c = \frac{H^3}{(18G/5)^{3/2}}\left(\frac{3}{4\pi\rho}\right)^2 \simeq 10^3 \odot. \qquad (32)$$

The thermal energy at $\Theta \simeq 10^2\,°K$ increases this value by a factor of order unity, its exact value depending on the density of the sphere.

A spherical cloud of sufficient mass will begin to contract. As in the non-magnetic case, we must ask whether contraction goes on indefinitely, and if so, whether isotropically, and finally, if and when fragmentation occurs. The contraction is determined by the forces acting, and in particular by the geometry of the gravitational field, itself a consequence of previous contraction. To begin, we ignore the dynamical causes of collapse, but discuss the dynamical consequences of different geometries of contraction. The flux of the field being conserved, we have

$$HR^2 = \text{constant}, \qquad (33)$$

R being now the radius of the equator defined by the direction of the field.

(a) Under <u>strictly isotropic collapse</u>, the radius along the field is again R, so that

$$M = \frac{4\pi}{3} \rho R^3, \tag{34}$$

and by (33)

$$H \propto \rho^{2/3}. \tag{35}$$

Hence

$$\mathfrak{M} = \frac{1}{6} \frac{H^2 R^4}{R} \propto \frac{1}{R} \tag{36}$$

and

$$-\Omega = \frac{3}{5} \frac{GM^2}{R} \propto \frac{1}{R}. \tag{37}$$

Thus if $-\Omega > \mathfrak{M}$ initially, so that collapse can start, then the assumption of indefinite isotropic collapse does not lead to any immediate conflict with the virial theorem, since $\mathfrak{M}$ and $-\Omega$ increase proportionately. This should be contrasted with the effect of a centrifugal contribution to kinetic energy, which in a cloud conserving its angular momentum increases like $1/R^2$, and so must ultimately halt isotropic collapse, as already noted in 2.1. Further, if the collapse is isothermal, the thermal energy stays constant and so becomes steadily a smaller fraction of the magnetic energy. Thus the minimum spherical mass that is gravitationally bound decreases towards the value (32) computed for zero thermal energy. However, isotropic change of scale cannot reduce the mass below this value, so that fragmentation is never possible, whatever the degree of spherical collapse.[25]

(b) <u>Cylindrical contraction</u> - the cloud contracts across the field but not along it. Then

$$H \propto \frac{1}{R^2} \propto \rho. \tag{38}$$

The lateral magnetic force density becomes of order

$$\frac{H^2}{4\pi R} \propto \frac{1}{R^5};\tag{39}$$

the lateral gravitational force density $\rho|\underset{\sim}{g}|$ is by Gauss's theorem

$$\simeq \frac{G(\pi R^2\rho)}{2\pi R}\rho = \frac{1}{2}G\rho^2 R \propto \frac{1}{R^3}.\tag{40}$$

Thus the increasing magnetic force stabilises the cylinder against indefinite collapse onto its axis. This is the property used by Chandrasekhar and Fermi in their model of a galactic spiral arm.[26] Since the magnetic force is increased more rapidly than the gravitational, it is clear that fragmentation will not be assisted by this type of contraction.

(c) Flow down the field towards a flattened distribution is unimpeded by the field. If ρ_0 and R_0 are the initial density and radius of the spherical cloud, then in the flattened state the density ρ and semi-thickness $\bar{z}$ satisfy

$$\rho\bar{z} = \rho_0 R_0,\tag{41}$$

while H stays equal to its initial value H_0.

Suppose now that the initially spherical cloud has a mass just above the critical value M_c (32). In neither the original nor in a more compressed spherical state could a smaller mass - say $\odot$ - separate out. In the roughly uniform massive cloud, the disruptive magnetic pressure of the field within a small spherical blob is balanced by the compressive effect of the external field—as measured by the surface term in (30). Should such a blob "try" to collapse spherically, the increasing internal magnetic force will no longer be balanced by the external pressure, so that collapse cannot proceed.

However, the situation is radically altered by longitudinal collapse, in which the density goes up but the field stays constant. For the condition that a sphere of radius $\bar{z}$ - the semi-thickness - should be gravitationally bound against magnetic disruption is by (31)

$$\frac{3}{5}G\frac{\left(\frac{4\pi}{3}\rho\bar{z}^3\right)^2}{\bar{z}} > \frac{1}{6}H_0^2\bar{z}^3,\tag{42}$$

where we note it is still H_0 that appears on the right. But by (41), this condition is identical with

$$\frac{3}{5} G \frac{\left(\frac{4\pi}{3}\rho_0 R_0^3\right)^2}{R_0} > \frac{1}{6} H_0^2 R^3, \qquad (43)$$

which is the condition, presumed satisfied, that the whole spherical cloud should be gravitationally bound. Hence after this longitudinal collapse, a much smaller spherical mass

$$\left(\frac{4\pi}{3}\rho_0 R_0^3\right)\left(\frac{\bar{z}}{R_0}\right)^2 \qquad (44)$$

can be gravitationally bound. Provided the collapse is not halted by thermal pressure, a ratio $\bar{z}/R_0 \simeq 1/30$ will enable spheres of stellar order to form.

It should be noted that for a spherical blob to separate out, the mass of the cloud that collapses into a disk must itself be above the critical value M_c (32) (for a prescribed ρ_0 and $\underset{\sim}{H_0}$). Thus suppose the cloud is just above M_c, but that only a fraction $\eta \ll 1$ of the mass collapses along the field into a disk, so that (41) is replaced by

$$\rho \bar{z} = \eta(\rho_0 R_0). \qquad (45)$$

Then a spherical blob of radius $\bar{z}$ will not be bound — a factor η^2 will appear on the left hand of (43). The same factor η^2 appears if one tries to make a sphere by collecting the matter in a sub-disk.

So far we have assumed that the cloud is initially a gravitationally bound sphere: we deduced the lower limit (32) to its mass, and then enquired how this is affected by different geometries of contraction. However, if the cloud has collapsed parallel to the field into a highly flattened structure, this lower limit is substantially reduced. Suppose the whole cloud approximates to a spheroid of radius R, height $2\bar{z}$, density ρ and mass M. Its volume is $\frac{4\pi}{3}R^2\bar{z}$ and its total magnetic energy is

$$\mathcal{M} = \frac{H^2}{8\pi} \times \frac{4\pi}{3}R^2\bar{z} = \frac{1}{6}H^2R^2\bar{z}. \qquad (46)$$

At a point within the disk, distant ϖ from the axis, the radial gravitational field is (cf. (4)) $\simeq \pi^2 G\rho\bar{z}\varpi/R$; hence this component contributes to the gravitational energy (19) the amount

$$\int_0^R \rho\varpi\left(\frac{\pi^2 G\rho\bar{z}\varpi}{R}\right)4\pi\varpi\bar{z}\,d\varpi = \frac{9\pi}{16}\frac{GM^2}{R}. \qquad (47)$$

(The contribution of the z-component is smaller by a factor $\bar{z}/R$ and is therefore dropped). Hence such a disk-like object exerts enough gravitation to overcome its magnetic pressure if

$$K = \frac{3}{5}\frac{GM^2}{R} \Big/ \frac{1}{6}H^2R^3 > \frac{16}{15\pi}\left(\frac{\bar{z}}{R}\right). \tag{48}$$

Written in this way, this condition relates the upper limit to the axial ratio $\bar{z}/R$ (assumed small in the computation) to the ratio K of the gravitational and magnetic energies of the cloud if it were spherical rather than disk-like. This quantity K is a constant parameter for the cloud - as already seen, it does not vary with R (provided the field is frozen in). If the cloud flattens indefinitely it can simultaneously shrink in radius R provided it retains its disk-like geometry, with $\bar{z}/R \simeq 15\pi K/16$. With the cloud temperature finite, thermal pressure will ultimately halt the flattening at $\bar{z}$ given by (23)

$$2\pi G \rho \bar{z}^2 \simeq c^2. \tag{49}$$

Still assuming the cloud stays as one structure, we find for the equilibrium radius — when both sides of (48) balance —

$$R \simeq \left(\frac{GM}{c^2}\right)\left(\frac{45\pi}{32}\right)K; \tag{50}$$

or

$$\frac{\Omega_s^2}{U\mathcal{M}_s} \simeq \frac{192}{675\pi} = \frac{1}{11}, \tag{51}$$

where the suffix S implies that the quantities are computed for a sphere of radius R.

As an example, consider a cloud of solar mass, which has flattened isothermally to this equilibrium state. Even if it is dense enough for thermal pressure (at 100°K) to be negligible, such a mass could not be gravitationally bound as a sphere, if its magnetic field has the "frozen-in" strength, $(\propto \rho^{2/3})$, associated with our standard values $H_0 = 10^{-6}$, $\rho_0 = 10 m_H$. In the bound, flattened state, however, we have $R \simeq 5\times10^{4}$, $\bar{z} \simeq 1.5\times10^{13}$, and $\rho \simeq 10^{-10}$. No doubt before such high densities could be reached, the increased opacity would tend to trap the heat of compression, so that the temperature would rise above its original values of $10^2\ {}^\circ K$ and slow up the contraction.

Further, the continued flattening of the cloud makes fragmentation possible: meaning not that spherical blobs can form, but that from the disk, sub-disks could form, each with magnetic pressure balancing gravity in two dimensions, and thermal pressure in the third. The point is that the magnetic field becomes a systematically smaller impediment, the greater the flattening parallel to the field; and the thermal pressure can always be balanced by an increase in the density, brought about by an adeqate collapse across the field. For suppose we prescribe the mass M that we wish to be gravitationally bound: (remembering that by hypothesis we are dealing with a cloud that does not have enough mass for any spherical blob to be bound, however much the cloud as a whole collapses along the field). Then by (32) and (48)

$$\frac{\bar{z}}{R} = \frac{15\pi}{16}\left(\frac{M}{M_c}\right)^2 . \qquad (52)$$

As $M = 4\pi \varsigma R^2 \bar{z}/3$, (49) yields

$$\frac{\bar{z}}{R^2} = \frac{2}{3}\frac{c^2}{GM_c} \Big/ \left(\frac{M}{M_c}\right), \qquad (53)$$

so that

$$R = \frac{45\pi}{32}\left(\frac{GM_c}{c^2}\right)\left(\frac{M}{M_c}\right)^3, \qquad (54)$$

and

$$\bar{z} = \frac{675\pi^2}{512}\left(\frac{GM_c}{c^2}\right)\left(\frac{M}{M_c}\right)^5 . \qquad (55)$$

Again, the rise in opacity would probably slow up the contraction of a small mass to its equilibrium state; but given an equilibrium temperature determined by balance between radiative absorption and emission — e.g. $10^2\ {}^\circ K$ — then the equations (54) and (55) fix the disk-like equilibrium. By forcing the mass to take up a disk-like structure, the magnetic field also ensures that the blob can reach mechanical equilibrium without having to acquire a high internal temperature - it never becomes anything like a star.

2.5 The dynamics of the collapse.

Suppose the cloud is spherical, and of a mass well above the critical mass. The essentially non-isotropic magnetic resistance is small, so that the spherical gravitational field keeps the cloud spherical during its collapse. However, if the mass is only slightly above M_c there will be a substantial dilution of the gravitational pull across the field, and a consequent preferential flow down the field. The increase in the field strength will therefore be less rapid than the $\rho^{2/3}$ law valid for isotropic collapse; and at first one expects that because the increase in the lateral gravitational pull is so much greater than the increase in the magnetic resistance, then nearly isotropic collapse will follow. However, the new (oblate) structure of the cloud yields a non-spherical gravitational field that acts preferentially down the field. For example, in a uniform oblate spheroid of eccentricity $\sqrt{3}/2$, corresponding to an axis ratio of 2, the relative acceleration towards the center of points on the axis is about 2.24 times as large as that of points on the equator.[15] As $e \to 1$ (strong flattening) this ratio behaves like $3/(1-e^2)^{1/2}$. Thus once the cloud has acquired a moderately oblate structure - whether or not due to an internal magnetic field - the gravitational field itself increases the flattening still further. Only when the cloud is opaque enough for its (essentially isotropic) thermal pressure to be comparable with gravity would the mass distribution (and hence the gravitational field) tend to revert as close as possible to isotropy.

If the sphere is below the critical mass, it cannot stay spherical and bound. The magnetic force acting laterally tends to disrupt the cloud against the resistance of the weaker gravitational pull; but down the field the gravitational pull is unresisted (temperature being low). Without more detailed work one cannot say which effect wins - whether the cloud disrupts, or collapses into a gravitationally bound disk described by (Ω). Preferential external compression would assist collapse into a disk.

As always, there will be present a field of density fluctuations of all wavelengths up to the cloud radius; again we phrase the fragmentation problem: "Which fluctuations are amplified against the increasing background density?" The Jeans problem in a uniform medium at rest, pervaded by a uniform field $\underset{\sim}{H}$, has been solved by Chandrasekhar and Fermi.[24] They find that the Jeans criterion is again unaltered except for waves travelling strictly across $\underset{\sim}{H}$. In the exceptional case, the critical length λ_H is related to the Jeans length by

$$\lambda_H^2 = \lambda_J^2 \left[1 + \frac{H^2}{4\pi\rho c^2} \right] . \qquad (56)$$

Again, we look upon these results as indicative of what we expect to result when the analogue of Hunter's analysis is worked out: i.e., we expect that sufficiently strong density fluctuations of scale well above the minima allowed by the Jeans-type analysis (or the virial theorem), will grow more rapidly than the background density. (However, it must be remembered that the result (56) depends on the medium being slowly varying in density in the direction of $\underset{\sim}{H}$. The problem could be redone in a flattened medium, or more simply, the virial theorem used to estimate the mass and structure of the sub-condensations to be expected).

For example, consider a spherical cloud (non-rotating), with a mass several times M_c, collapsing spherically under its essentially undiluted gravitational field. We expect it to fragment into sub-masses, each of the same order as M_c though necessarily somewhat larger. It is this imprecision in the value of the mass of the fragments that prevent one from asserting definitely that the fragment will be markedly oblate, because of the presence of a magnetic energy comparable with the gravitational. However, if they are oblate, then they will flatten still further; following the same line of argument, we should expect smaller masses - given roughly by (44) - to separate out. These small blobs will also have a magnetic energy comparable with their gravitational energy.

However, since the field does not interfere with flow parallel to itself we may expect to form, within each gravitationally bound blob, high density layers across the field, of thickness of the order of the instantaneous Jeans length. This local increase of density, without a corresponding increase in $\underset{\sim}{H}$, yields a local gravitational field that is able to distort the local magnetic field: in fact we arrived at a sub-structure as described by our equations (54) and (55). What one cannot derive are spherical, gravitationally bound blobs of low mass: we have already noted that bound spheres of systematically lower mass can arise only if the whole cylindrical height of a cloud of mass $> M_c$ collapses towards a disk. Presumably, once the sub-sphere as a whole becomes dense and therefore opaque enough for its temperature to rise, so that it becomes effectively a proto-star, then such stratification would be smoothed out.

A proper dynamical theory should again predict the energy of the ultimate sub-condensations. However, in view of the wholly improper neglect of angular momentum, it is hardly worth pursuing this topic here. We have gained enough from studying magnetism and rotation acting alone to attempt to consider their joint effect.

The ultimate fate of a cloud that has too little mass to be bound as a sphere, but has become a bound magnetic disk, is an interesting problem. We have seen that any fragmentation will be into sub-disks, which will never shine as stars. The effects of possible Raleigh-Taylor instability need to be explored; but again any realistic studies must take account of rotation.

2.6 The coupling between gas and magnetic field.

So far we have assumed without question the "freezing of the field" into the moving gas. This is a well-known kinematic consequence of the equation of electromagnetic induction

$$\nabla \wedge \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{H}}{\partial t} \tag{57}$$

and the form of "Ohm's law" for a simple moving conductor

$$\underset{\sim}{E} + \frac{\underset{\sim}{v} \wedge \underset{\sim}{H}}{c} = \frac{\underset{\sim}{j}}{\sigma} \tag{58}$$

in the limit as the conductivity $\sigma \rightarrow \infty$.[27]

In a fully ionised gas (plasma) the analogue of Ohm's law is similar to (58), except for extra terms on the right (the Hall and electron pressure terms)[27]. Their effect is usually (though not always) small in cosmical applications. Further, the length-scales of cosmical problems are so large that the current density $\underset{\sim}{j}$ required by Ampère's law

$$\underset{\sim}{j} = \frac{c}{4\pi}\nabla \wedge \underset{\sim}{H} \tag{59}$$

to maintain fields of given strength are very much smaller than the corresponding terrestrial currents maintaining small-scale fields. Thus because of the moderately high conductivity, the Ohmic term $|\underset{\sim}{j}/\sigma|$ is usually much smaller than the "dynamo" term $|\underset{\sim}{v} \wedge \underset{\sim}{H}/c|$ even for slow velocity fields (such as circulation speeds inside stars): the "magnetic Reynolds number" is high, and the infinite conductivity approximation valid. Only in singular regions will the Ohmic term dominate: for example, in the neighbourhood of O-type neutral points, where there is slow but inevitable destruction of the magnetic flux (Cowling's theorem)[27]; or when oppositely directed field lines become arbitrarily close, so that they diffuse and annihilate each other.

However, the gas in our problem is in general only partially ionised. In a standard $H\,I$ cloud there is one metallic ion present to about 10^4 neutral hydrogen atoms. In the hypothetical primeval galactic medium of nearly pure atomic (i.e. non-molecular) hydrogen, the cloud is kept at near $10^4\,^\circ K$ by a balance between compressional heating, and loss by collisional ionisation and radiative recombination. However, even here it is unlikely that the cloud will be even approximately fully ionized (though there will be a far higher fraction of ions than in an $H\,\underline{I}$ cloud). Hence the equations valid in a plasma cannot strictly be applied to either case, and it is not immediately clear that the freezing of the field is a good approximation.

In fact, in a lightly ionised gas the drift of the magnetic field through the matter is far more rapid than in a plasma, unless the field strength is very low.[25] The freezing of the field into the plasma component of the gas remains an excellent approximation in most cosmical problems. But the magnetic force acts directly only on the plasma, and makes it diffuse through the neutral gas; quasi-equilibrium is reached when the magnetic force is balanced by the plasma-neutral friction acting on this drift velocity. Approximately

$$\frac{\underset{\sim}{j} \wedge \underset{\sim}{H}}{c} + n_i \gamma \left(\underset{\sim}{v}_H - \underset{\sim}{v}_i \right) = 0, \tag{60}$$

where $\underset{\sim}{v}_H$ and $\underset{\sim}{v}_i$ are bulk velocities of the neutral hydrogen and the plasma, n_i is the plasma density, and the coupling constant γ is such that the mean frictional drag on an ion is

$$\underset{\sim}{F}_{iH} = \gamma \left(\underset{\sim}{v}_H \quad \underset{\sim}{v}_i \right) = n_H \, \sigma_{iH} (v_H)_{\circledR} \, m_H \left(\underset{\sim}{v}_H - \underset{\sim}{v}_i \right). \tag{61}$$

Here n_H is neutral hydrogen density, σ_{iH} the cross-section for ion-hydrogen collisions, and m_H the mass of the hydrogen atom. Thus given n_H and n_i and the temperature of the cloud we can compute from (60) the rate at which the magnetic force drives the plasma through the neutral gas. Since the equation of motion of the electrons (or the ions) reduces to

$$\underset{\sim}{E} + \frac{\underset{\sim}{v}_i \wedge \underset{\sim}{H}}{c} = \text{small terms}, \tag{62}$$

we have that (60) yields simultaneously the drift of the lines of $\underset{\sim}{H}$ through the cloud as a whole - in general a much larger contribution than the slow diffusion due to the Ohmic field. An exception is the case of the force-free field, in which $\underset{\sim}{j}$ is parallel to $\underset{\sim}{H}$; such a field is irrelevant to most of our problems.

Equivalently, the rate of dissipation of magnetic energy is far higher than in a plasma. From (62), we have approximately

$$\underset{\sim}{j} \cdot \underset{\sim}{E} = \left(\frac{\underset{\sim}{j} \wedge \underset{\sim}{H}}{c}\right) \cdot \underset{\sim}{v_i} = \left(\frac{\underset{\sim}{j} \wedge \underset{\sim}{H}}{c}\right) \cdot (\underset{\sim}{v_i} - \underset{\sim}{v_H}) + \left(\frac{\underset{\sim}{j} \wedge \underset{\sim}{H}}{c}\right) \cdot \underset{\sim}{v_H} . \qquad (63)$$

On the left we have the rate at which the electromagnetic forces do work on the charged particles: if positive it represents a net loss of magnetic energy; if negative, a storing of magnetic energy. Since $\underset{\sim}{V_H}$ is effectively the bulk velocity of the gas as a whole (the plasma component being slight), the second term on the right is the rate at which the magnetic body force does work on the gas. The other term is by (60)

$$\left|\frac{\underset{\sim}{j} \wedge \underset{\sim}{H}}{c}\right|^2 \Big/ n_i \gamma = \underset{\sim}{j_\perp}^2 \Big/ \frac{n_i \gamma}{c H^2} = n_i \gamma \left(\underset{\sim}{v_i} - \underset{\sim}{v_H}\right)^2 , \qquad (64)$$

where $\underset{\sim}{j_\perp}$ is the component of current across $\underset{\sim}{H}$. This term is the rate of dissipation of energy by friction: for currents $\underset{\sim}{j_\perp}$ it can be regarded as due to an anomalous resistivity that is $\propto H^2/n_i$.

In the problem of a gravitationally collapsing magnetic cloud, the velocity $\underset{\sim}{V_H}$ is essentially the free-fall velocity, diluted a little by the magnetic resistance (transferred via the plasma-neutral friction). If $|\underset{\sim}{v_i} - \underset{\sim}{v_H}| \ll |\underset{\sim}{v_H}|$, then the plasma-plus-field effectively moves with the neutral gas; even though the production of heat (64) is far greater than in a plasma, it is small compared with the rate $-(\underset{\sim}{j} \wedge \underset{\sim}{H}/c) \cdot \underset{\sim}{V_H}$ at which the gravitationally driven flow $\underset{\sim}{V_H}$ pumps energy into the magnetic field. On the other hand, if $|\underset{\sim}{v_i} - \underset{\sim}{v_H}| \gg |\underset{\sim}{v_H}|$, the essentially disruptive magnetic force drives the plasma out, and by inductive coupling the field moves out with it. A new quasi-equilibrium state will be reached, in which the magnetic field lines have much less curvature, and so exert a much weaker force, balanced by the frictional force due to neutral gas falling across the field.

Whether this uncoupling between matter and field occurs depends on the strength of the frictional parameter ($n_i \gamma$) - i.e, for a given density, essentially on (n_i/n_H). Taking 10^{-16} for σ_{iH}, Mestel and Spitzer[25] found that a "normal" value of 10^{-4} for n_i/n_H in an $H\,I$ cloud is too large by about 100 for the drift of plasma to approach the free-fall speed. But in a dense dust cloud the plasma density may drop rapidly - mainly by attachment to dust grains - to a small fraction of its normal value, if the galactic ionizing radiation is extinguished at the cloud surface by the dust. However, the recent calculations by Osterbrock[28] yield a value $\simeq 10^{-14}$ for σ_{iH}; while cosmic ray ionization tends to keep n_i from falling. It seems therefore uncertain whether or not substantial loss of magnetic flux by the cloud can occur during its gravitational collapse. If it does, the star formation problem reduces to the fragmentation problem, with or without angular momentum, but with no primeval magnetic field of any strength to impede sub-condensation. Whether the field could subsequently assist in removing angular momentum from a proto-star depends partly on the stage at which coupling between field and matter is re-established.

Both this process, and the preferential flow already discussed, lead to an increase in ρ, without the increase in H $(\propto \rho^{2/3})$ associated with spherical collapse of a frozen-in field (the result that kept the minimum mass above M_c (2:32)). However, whereas sufficient loss of magnetic flux results in the magnetic field becoming a small perturbation, with preferential flow the frozen-in magnetic field still plays an important role in the fragmentation problem; for if the temperature stays low, the field determines the lower limit to the mass that can separate out at any epoch (cf 2.5). Thus the blobs that form all have substantial magnetic energies, comparable with their gravitational energies. It is possible that during the contraction to the main sequence, internal convective turbulence may succeed in destroying most of this magnetic energy; but even if the star conserves its primeval field, it does not mean that we have a contradiction with the observed fact that most stars have weak external fields. An inexorable internal meridian circulation, driven e.g. by centrifugal force, will tend to keep the star's general field beneath the surface.[29] An initial external part would ultimately be detached from the internal field by Ohmic diffusion, and would be blown away by the stellar wind. It is significant that we have evidence - from the polar plumes - of a general solar field only in regions where we have no evidence of even moderate circulation speeds; whereas in the sunspot zones, sub-photospheric circulation is probably rapid enough to prevent the appearance above the surface of even a weak external general field. To sum up: it may very well be that in order that Type I stars can form at all, we may have to appeal to preferential flow down the field lines, rather than the Mestel-Spitzer process of flux-loss; but we do not have the impossible task of building a star by flow down the field lines, such that the internal field has the low value indicated by the observed solar external field.

Relative drift of plasma and neutral gas may be important in other problems. The flattened magnetic disks of 2.4 could revert to a spherical structure if there were a systematic loss of flux in a reasonable time; as they cannot contract further across the field, there is no question of the loss of flux having to occur within the free-fall time. However, the high densities in the flattened state - which is approached in the free-fall time - probably prevent any substantial drift even in 5×10^9 years. We shall find that in eccentric regions - e.g. pinched zones - drift may become large even though it is negligible over the bulk of the cloud.

Finally, we note that even if Type I stars do form from clouds which have lost most of their magnetic flux by the Mestel-Spitzer process, it is clear that clouds of pure atomic hydrogen - postulated for the matter from which the oldest Type II stars formed - cannot lose their flux in this way. In HI clouds the processes by which the temperature is kept low - molecular excitation, thermal radiation from dust grains - are essentially separate from the processes of ionization. But in a primeval cloud of atomic hydrogen, gravitational collapse automatically keeps enough hydrogen ionized so that the compressional energy generated can be radiated away. Thus there is always enough plasma to keep the field coupled effectively to the cloud as a whole; if a strong galactic field existed when the old Type II stars were formed, then preferential flow seems the only way by which small masses could condense.

3. THE EVOLUTION OF A ROTATING MAGNETIC CLOUD

We now attempt a more realistic, but also much more difficult problem than any discussed so far - that of a cloud with a high angular momentum and a strong magnetic field. There is clearly ample scope for complication in the different magnetic structures one can postulate, and in the inclination of the axis of the field (assumed large-scale) to the rotation axis. We shall discuss only two highly idealised models. In the first, the cloud is roughly spherical and has a field which is more or less parallel to the axis. This model can be discussed in some analytical detail because of its having an axis of symmetry. In the other model (discussed only qualitatively), the field lines are again straight within the cloud, but they lie in planes perpendicular to the rotation axis. Since the galactic magnetic field is believed to lie mainly in the galactic plane and therefore prependicular to the probable direction of rotation, the second model seems the better paradigm, at least for star formation in the present epoch.

Since we are interested in magnetic braking, we begin by assuming that the cloud field lines initially extend out to "infinity". However, the structure of the field is not something that can be prescribed but is determined by the hydromagnetics of the problem; one of our tasks is to estimate when field lines detach themselves from the local galactic field and form closed loops, thus sharply cutting down the rate of magnetic braking.

A very strong turbulent field within a cloud would alter the whole nature of the problem if it could twist and tangle the field sufficiently to destroy it by Ohmic dissipation. We cannot be certain that this has not happened during the formation of some galactic clouds. The magnetic field is then not there to be either a hindrance or a help; the star formation problem now consists of getting over the angular momentum difficulty by agglomeration of matter into a body of low rotatory but high orbital angular momentum. (cf. 2.3). However, if we start with a cloud with only moderate turbulence, and with a mass well above the critical mass M_c (2.3) for gravitational binding against magnetic resistance, then it is difficult to see why the kinetic energy generated in the subsequent collapse should become small scale turbulent energy: rather one expects the cloud to fragment into randomly moving sub-clouds, each with an internal magnetic field of energy comparable with the gravitational energy of the sub-cloud. This is the type of system that we discuss. Within each cloud the strong magnetic field controls the turbulent eddying, rather than being tangled by the turbulence.

3.1 The magnetic axis parallel to the angular momentum vector:collapse and fragmentation.

For definiteness, consider a cloud which is roughly spherical, of a mass M above the critical mass M_c and with initially negligible centrifugal forces. Then if $M \gg M_c$ we have seen that the magnetic dilution of the lateral gravitational field is small, so that the initial collapse is spherically symmetric, and no fragmentation can occur. But the increasing centrifugal force - assuming strict conservation of angular momentum - ultimately halts contraction across the axis of the system, and forces preferential flow down the field, and ipso facto parallel to the rotation axis.

If the cloud has a mass just above M_c, then the reduction in lateral gravitation by the field will in any case cause initial preferential flow, which we saw in 2.5 is accentuated by further flattening. The effect of introducing a rotation vector parallel to $\underset{\sim}{H}$ is to ensure that the flattening takes place, even if $M \gg M_c$ and the cloud is initially spherical.

This flow towards the equator enables the cloud to fragment. We saw in Chapter 2 that if all of a cloud of mass $\gg M_c$ collapses into a flattened state, then systematically smaller spherical blobs can separate out and begin to contract against the resistance of their internal magnetic fields. At the same time, the centrifugal force continues to "hold up" the cloud as a whole, but it no longer prevents spherical sub-condensations from forming (though we note again the mathematical difficulties associated with rotational shear). When we ignored rotation, we appealed to Hunter's work[9] to justify our picture of smaller blobs separating out more quickly than the collapse of the whole cloud. However, with a strong centrifugal field collapse in two dimensions is in any case halted:as emphasised by McCrea,[23] rotation helps fragmentation by preventing the cloud as a whole from collapsing into a small sphere.

However, it is one thing to have the flattened cloud break up into blobs, with or without substantial energy of z-motion; it is another matter for blobs of stellar mass to be able to reach the main sequence. With a strong magnetic field effectively controlling any random eddying motion, it is difficult to believe that formation of blobs with accidentally low spin is the answer tothe problem. But we have seen that if there is systematic removal of angular momentum during either the early stages of star formation, or during the approach of an opaque proto-star to the main sequence, then the difficulty can be overcome. Since we are concerned with the collapse and fragmentation of a magnetic cloud, it is natural to ask whether the field is able in these early stages to transport much angular momentum. If the cloud contracts, conserving its angular momentum, its spin increases, and the part of a field line within the cloud will rotate more rapidly than the outer part. A toroidal magnetic component is generated, and the resulting torque attempts to equalise the angular velocities of the cloud and the external matter. Later we must discuss the structure of the field outside the contracting cloud: in particular, we must decide which field lines, if any, remain "infinite", and which systematically detach themselves from the galactic field. For the moment we take the most favourable case, and assume the field lines stay infinite, so that there is a continuous transport of angular momentum to infinity. If the cloud were at rest, its spin would steadily be reduced; but since the cloud is contracting, and so tending to increase in spin, the problem is one of relative time-scales.

To estimate the best possible rate of magnetic braking, we suppose that outside the cloud the density is low and the field reasonably strong, so that hydromagnetic waves travel rapidly, and keep the surface of the cloud rotating with the background. The non-uniformity in rotation between the surface and the center is ironed out in a time

$$\frac{R}{V_A} = \frac{R}{(H/4\pi\rho)^{1/2}} \simeq \frac{R^2\,(4\pi\rho)^{1/2}}{(HR^2)^{1/2}}, \tag{1}$$

V_A being the Alfvén speed. By comparison, the time of gravitational collapse is roughly

$$\frac{R}{(2GM/R)^{1/2}} \simeq \frac{1}{(8\pi G\rho/3)^{1/2}}, \tag{2}$$

and the ratio of (1) to (2) is

$$\frac{R^3\,(4\pi\rho)^{1/2}}{(HR^2)^{1/2}}\,(8\pi G\rho/3)^{1/2} \simeq \left(\frac{GM^2}{R}\Big/\frac{H^2R^3}{3}\right)^{1/2}, \tag{3}$$

which is of the order of the square root of the ratio of gravitational to magnetic energy. Thus if a gravitationally bound mass has about as much magnetic energy as is allowed, our order of magnitude argument suggests that infinite field lines may be able to keep the spin low during the contraction. But since the free-fall speed and the Alfvén wave speed are comparable, one is forced to do a more accurate calculation to reach a definite result.

It should be noted that too efficient a loss of angular momentum could be an embarrassment. First of all, we know that O and B stars reach the main sequence with nearly the upper limit of angular momentum, and so we do not want all contracting globules to be kept rotating at e.g. 10^{-15}/sec until too high densities. (It is far more plausible that the sharp drop in spin below Type F is due to a special deceleration process for low mass stars, than that initially slowly rotating O and B stars have been accelerated.)

Secondly, in our particular model, we have appealed to centrifugal force in order to be certain that there will be preferential flow down the field, so that small globules can form. Too efficient a removal of angular momentum could enable the cloud or globule to contract isotropically - at least if its mass is well above M_c (2.32).

3.2 Magnetic deceleration of a rotating collapsing mass.

We adopt the following highly idealised model. The cloud is uniform and spherical, and with a uniform internal magnetic field parallel to the axis of rotation. Outside the cloud, the field is nearly radial (except near the equator), its value being given by the continuity of the normal component. The tangential discontinuity implies a surface current, and a locally infinite volume force, while the uniform field is curl-free, and so exerts no force. These unphysical features are avoided in the more accurate study of 3.3; but the present model is adequate for the magnetic braking problem.

If magnetic braking is going to be efficient, then centrifugal force must be small, and so does not distort the sphere. We shall assume that the sphere collapses isotropically - i.e., we ignore also the essentially non-isotropic magnetic force. (Since we expect this force to be sizeable in the case of interest, this assumption is dubious; but the results are likely to be similar for cylindrical collapse). We take for the radius of any mass shell m at time t

$$\frac{r(m,t)}{r_0(m)} = f(t) = \left[1-(6\pi G\rho_0)^{1/2}t\right]^{2/3} = (1-\gamma t)^{2/3}, \qquad (4)$$

the integral of

$$\ddot{r} = -\frac{Gm}{r^2} \qquad (5)$$

with the initial condition

$$\dot{r}_0^2(m,0) = \frac{2Gm}{r_0(m)}. \qquad (6)$$

It is convenient to use as a time-variable

$$\tau = -\log(1-\gamma t) = -\frac{3}{2}\log s(t) = -\frac{3}{2}\log\left(\frac{r}{r_0}\right). \qquad (7)$$

Because of the assumption of uniform density, the cloud contracts homologously - γ is independent of m, and τ is a uniquely defined variable.

To discuss the interaction between the magnetic and rotation fields we use cylindrical polar coordinates (ϖ, ϕ, z). The torque equation is

$$\frac{\partial}{\partial t}(\Omega\varpi^2) = \frac{\mathbf{H}\cdot\nabla\left(\frac{\varpi H_\phi}{4\pi}\right)}{\rho}, \qquad (8)$$

where Ω is again the angular velocity, and $\partial/\partial t$ is a derivative following the motion. The left-hand side is the rate of change of angular momentum of a moving particle of unit mass; the right is the magnetic torque density, reduced to unit mass by the factor $1/\rho$.

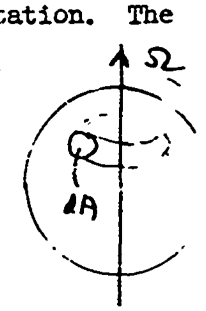
The hydromagnetic equation and the continuity equation yield

$$\frac{\partial}{\partial t}\left(\frac{H_\phi}{\rho\varpi}\right) = \frac{\mathbf{H}\cdot\nabla\Omega}{\rho}. \qquad (9)$$

This can be derived directly by writing the familiar equations

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla\wedge(\mathbf{v}\wedge\mathbf{H}), \qquad \frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}) \qquad (10)$$

in cylindrical polars. Physically (9) can be interpreted as follows. In the absence of non-uniform rotation, the toroidal magnetic field changes just by virtue of the non-rotatory motion. Consider a torus of cross-section dA its axis being the axis of rotation. The flux of H_ϕ through the torus is ($H_\phi\, dA$); the mass of the torus is $2\pi\rho\varpi\, dA$. If the torus following the motion, and the rotation is uniform, then



$$\frac{H_\phi}{2\pi\rho\varpi} = \text{flux of } H_\phi \text{ per unit mass} = \text{constant}. \qquad (11)$$

Non-uniform rotation generates a toroidal component from a poloidal H_p at the rate $\varpi(\underline{H}.\nabla)\Omega$, so that the rate of change of the flux of H_ϕ per unit volume is $(\underline{H}.\nabla)\Omega/2\pi$; per unit mass $(\underline{H}.\nabla\Omega)/2\pi\rho$; whence (9).

By our prescribed motion (4),

$$\varpi = \varpi_0/f, \qquad z = z_0/f, \qquad \rho = \rho_0/f^3, \qquad H_p = H_z = H_0/f^2, \tag{12}$$

so that (8) and (9) become

$$\frac{4\pi\rho_0\varpi_0}{H_0}\frac{\partial}{\partial t}(\Omega f^2) = \frac{1}{f}\frac{\partial}{\partial z_0}(H_\phi f^2) \tag{13}$$

and

$$\frac{\partial}{\partial t}(H_\phi f^2) = \frac{1}{f}\frac{\partial}{\partial z_0}(H_\phi f^2). \tag{14}$$

Together they yield

$$\frac{\partial^2\Omega}{\partial z_0^2} = \frac{1}{V_A^2}\frac{\partial}{\partial t}\left[f\frac{\partial}{\partial t}(\Omega f^2)\right], \tag{15}$$

where

$$V_A^2 = \frac{H_0^2}{4\pi\rho_0} = (\text{Alfvén speed})^2. \tag{16}$$

With the new time variable (7), and a non-dimensional coordinate

$$Z = \frac{\gamma}{V_A} z_0, \tag{17}$$

our equation reduces to

$$\left(\frac{\partial^2}{\partial\theta^2} - \frac{7}{3}\frac{\partial}{\partial\theta} + \frac{4}{3}\right)\Omega = \frac{\partial^2\Omega}{\partial Z^2}. \tag{18}$$

This must be solved subject to the boundary conditions:

(i) if $\bar{Z}$ is the value of Z at which the field line considered meets the edge of the sphere, then

$$\Omega = \Omega_\infty \quad \text{for all } v;$$

(ii) at $v = 0$

$$\Omega = \Omega_0(z) \qquad H_\phi = [H_\phi(z)]. \tag{19}$$

- prescribed initial values.

If these initial functions are even in z, then Ω will be even — $\partial\Omega/\partial z = 0$ at $z = 0$ for all v.

As a simple example, take $H_\phi = 0$ and $\Omega = \Omega_\infty$ at $v = 0$.
Under contraction shear develops at the surface of the cloud, and hydromagnetic waves are set up. The solution involves the natural parameter ζ where

$$\frac{9\pi^2 v_A^2}{\bar{Z}^2\gamma^2} = \zeta^2 \simeq \left[\frac{\text{Alfvén speed}}{\text{free-fall speed at edge}}\right]^2; \tag{20}$$

v_A/γ = distance travelled by an Alfvén wave in the free-fall time. If $\zeta > 1$, the angular velocity at time v is

$$\frac{\Omega(v,z)}{\Omega_\infty} = 1 + \frac{32}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)}\frac{e^{(7/6)v}\sin\left\{[(2k+1)^2\zeta^2-1]^{1/2}\frac{\zeta v}{6}\right\}}{[(2k+1)^2\zeta^2-1]^{1/2}}\cos\left(k+\tfrac{1}{2}\right)\frac{z\pi}{\bar{Z}}, \tag{21}$$

whereas in the absence of the field,

$$\frac{\Omega(v,z)}{\Omega_\infty} = e^{(8/6)v} = \frac{1}{f^2}. \qquad (\quad)$$

Thus the angular velocity oscillates with an amplitude roughly proportional to

$$\frac{e^{(7/6)v}}{\zeta} \propto \frac{1}{\zeta f^{7/4}}$$

The ratio of the maximum centrifugal force to gravity

$$\frac{\Omega^2 r^3}{G m} \propto \frac{1}{\zeta^2} \frac{1}{f^{1/2}} = \frac{e^{(2/3)\nu}}{\zeta^2}, \tag{24}$$

instead of the ($1/f$) law that holds under strict conservation of angular momentum. Thus if, under angular momentum conservation, isotropic collapse would be halted, e.g., after a contraction to $1/10$ of the initial radius, then if $\zeta > 1$ the cloud can contract by a factor $1/100\,\zeta^4$ before centrifugal balance is achieved.

If $\zeta < 1$ the dominant term in the solution is $\propto e^{(\nu/6)(7+\sqrt{1-\zeta})}$, so that if $\zeta \ll 1$ the magnetic field is quite negligible.

It appears that if the field is strong, there is a continual interchange of angular momentum between cloud and external matter, and that the crucial parameter (24) still increases with time, though more slowly than when $H = 0$. A sufficiently great permanent loss of angular momentum can occur only if ζ is very large. But ζ cannot be prescribed arbitrarily - we have seen that (with $\Sigma > R$) its order of magnitude is at most unity (cf. 2.4) One would like to see a more detailed discussion of the problem - with different initial condition, and a more thorough treatment of the boundary condition - but it seems unlikely that the deceleration can be much increased: one would think that the boundary condition $\Omega = \Omega_0$ is the most favourable.

As an example, suppose $\zeta \sim 1$, and so the cloud of mass $10^3\,\odot$ collapses to $R/100$ instead of $R/10$, after which centrifugal force makes it flatten into a spheroid with axial ratio ϵ, and then fragment into blobs of mass $M\epsilon^2$ and radii ϵR. Perhaps the frozen-in fields of these fragments in their turn remove some spin angular momentum from the orbiting fragments. A blob of mass $M\epsilon^2$ has a centrifugal parameter $\Omega^2/\pi G \rho \simeq \epsilon$; hence again taking $\zeta = 1$, the most that magnetic braking is likely to do is allow it to reach a radius $\epsilon^2(\epsilon R)$ (instead of ϵ (ϵR)) before in its turn it achieves centrifugal balnce. If the mass of the blob is to be of stellar order, $\epsilon \simeq 1/30$, and the radius it can reach before flattening is $\simeq 3\times 10^{-5} R$, which is still several orders of magnitude above main sequence radii.

It does not, then, seem likely that straightforward magnetic braking can play a decisive role during the free-fall of a cloud. However, once the fragment becomes opaque, it will contract far more slowly. The details of the theory will not be applicable, as the law of contraction is different from (4); but we may expect that a parameter analogous to (20) will appear. If R_f is the radius of a fragment of mass M_f, then the parameter will be approximately

$$V_A^2 / \dot{R}_f^2 . \tag{25}$$

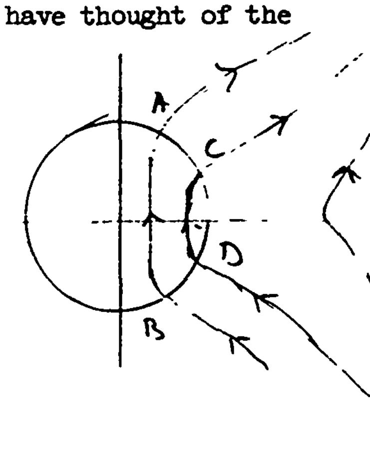
Assuming Kelvin-Helmholtz contraction (i.e. ignoring Hayashi convection), we have not only that in a strongly magnetic globule (25) is initially much greater than unity, but that also it increases with time; for

$$\dot{R}_f \simeq \left(\frac{L_f}{G M_f^2}\right) R_f^2 \tag{26}$$

where L_f is the luminosity, a slowly varying function of R_f. Thus at first sight it appears that we could relax considerably the stringent condition that the internal field be as large as possible, and still get considerable magnetic braking during this slow contraction phase.

Unfortunately, new difficulties arise. We have already noted (2.6) that once a globule becomes opaque, the adjustment of its thermal field to arbitrary non-spherical perturations (centrifugal and magnetic force) results in general in large-scale internal circulation. The circulation speeds, though slow over the bulk of the mass, become quite fast at low densities near the surface; if the flow persists, it will detach the internal and external parts of the field. The external part, being no longer anchored in the star, will be easily lost - e.g. blown away by a stellar wind. In the final quasi-static state a very few of the lines of force of the general field will leak out - their number depending on the magnetic Reynolds number of the circulation. A weak external field is no strong argument against a strong internal field — the relic of the local galactic field in the matter from which the star was born; but equally a strong internal field cannot be employed to brake the star if its lines are nearly all confined within the star.

As even more serious objection, which applies whether the cloud or globule is transparent or opaque, is that the assumption of infinite field lines is unlikely to be valid for long during the contraction. We have pictured the cloud as a local condensation in a region of roughly uniform field. The cloud collapses, dragging the field with it; the local distortion in the field extends beyond the "edge" of the cloud, but far enough away the field is effectively the original uniform field. Because we have thought of the field as strictly frozen-in to the gas, we have not envisaged any snapping of field lines, which have stayed "infinite". This is crucial for the magnetic braking problem, in that hydromagnetic waves can then travel rapidly far from the cloud, and we are justified in using the boundary condition $\Omega = \Omega_0$.



It might appear that this question of the existence of finite or infinite field lines is of no particular importance for this model, since in any case we have cast serious doubt on the efficiency of magnetic braking by a general magnetic field in either the free-fall or Kelvin-Helmholtz stages. However, the parameter ζ (20) contains the length $\bar{z}$ — half the length of the segment within the cloud of the field line considered. For most of the field lines - e.g. BA in the figure — $\bar{z} \simeq$ the radius R ; but those like DC are much shorter. The hydromagnetic waves do not have far to travel, and the appropriate value of ζ is much larger; hence provided the line DC extends to infinity, the theory predicts a much lower angular velocity for DC than for BA. But a sufficiently sharp decrease in Ω in planes perpendicular to the axis, is likely to be Rayleigh unstable, energy being released when neighbouring cylindrical shells are interchanged. One could thus imagine the cloud to lose angular momentum by hydromagnetic action along DC, while interchange instability continually replenishes the angular momentum of the DC neighbourhood from that of the rest of the cloud. The net loss of angular momentum could be much greater than by direct hydromagnetic action along lines such as BA. The structure of the field is therefore an important question, at least for the free-fall stage. We now study in detail the structure of the field, to see when the assumption of freezing of the field breaks down, and snapping of field lines takes place.

3.3 The magnetic field of a contracting cloud.

We study the following simple model. A medium of uniform density ρ_i, and pervaded by a uniform field $\underset{\sim}{H}_i$, is imagined to undergo a non-homologous contraction, yielding the spherically symmetric density field

$$\rho_0(r_0) = \rho_i + \rho_c \exp\left[-(r_0/R_0)^2\right], \tag{27}$$

where r_0 is the radial distance from the center of contraction; the central density is $(\rho_c + \rho_i) \simeq \rho_c$ if $\rho_i/\rho_c \ll 1$, and R_0 is a measure of the "radius" of the cloud. In terms of a non-dimensional radius $x_0 = (r_0/R_0)$ the new position of an element of mass is related to its original position $x_i = (r_i/R_0)$ by the conservation of mass:

$$\rho_i x_i^2 \, dx_i = \rho_0(x_0) x_0^2 \, dx_0, \tag{28}$$

or

$$\left(\frac{x_i}{x_0}\right)^3 = \frac{\bar{\rho}_0(x_0)}{\rho_i} = 1 + \frac{\rho_c}{\rho_i}\left(\frac{3\int_0^{x_0} u^2 e^{-u^2} du}{x_0^3}\right), \tag{29}$$

where $\bar{\rho}_0(x_0)$ is the mean density within x_0. Near the center $(x_0 \ll 1)$,

$$\frac{x_i}{x_0} \simeq \left(1 + \frac{\rho_c}{\rho_i}\right)^{1/3} \tag{30}$$

- a homologous contraction to the new, approximately uniform density. Far enough away - $x_0 \gg \rho_c/\rho_i$ - the change in the mean density within x_0 is negligible, and $x_i \simeq x_0$. In between, $1 < x_i/x_0 < (1 + \rho_c/\rho_i)^{1/3}$.

Once a massive enough cloud has been formed it will begin to contract; if magnetic and centrifugal forces could be ignored, it would fall freely, with the radius of a mass sphere at time t given by (cf. 4)

$$\frac{r}{r_0} = \frac{x}{x_0} = \left[1 - [6\pi G \bar{\rho}_0(x_0)]^{1/2} t\right]^{2/3}, \tag{31}$$

where again we have selected for simplicity the special solution for which

$$(\dot{r}^2)_{t=0} = \frac{2G\, m(r_0)}{r_0} \tag{32}$$

for all r_0. In general, whatever the initial radial velocity field, the inner parts with systematically higher mean densities will have a higher relative acceleration, so that the gravitational field tends to accentuate the mass concentration to the center, as illustrated by (31). Thus although strictly one should follow the motion, and so determine the evolving density field, we may regard the model (27), with systematically increasing ρ_c and decreasing R_0 as describing the effect of the spherical gravitational field at different epochs.

In the initial state, the uniform magnetic field is described by a Stokes stream-function

$$P_i(r_i, \theta) = -\frac{1}{2} H_i r_i^2 \sin^2\theta, \tag{33}$$

so that

$$H_r = -\frac{1}{r_i^2 \sin\theta}\frac{\partial P}{\partial \theta} = H_i \cos\theta,$$
$$H_\theta = \frac{1}{r_i \sin\theta}\frac{\partial P}{\partial r} = -H_i \sin\theta. \tag{34}$$

P is constant on field lines; $2\pi P$ is the total magnetic flux crossing the equator between the origin and r_i. Under the spherical condensation (27), the point (r_i, θ) becomes (r_0, θ); since the field lines are frozen in to the moving matter, we may compute the distorted field by substituting for r_i in (33), and then using (34) with $P_i(r_i, \theta)$ replaced by $P_0(r_0, \theta)$. We find

$$H_r = H_i \cos\theta \left\{ \frac{\overline{\rho_0(x_0)}}{\rho_i} \right\}^{2/3},$$

$$H_\theta = -H_i \sin\theta \left\{ \frac{\overline{\rho_0(x_0)}}{\rho_i} \right\}^{2/3} \left\{ \frac{\rho_0(x_0)}{\overline{\rho_0(x_0)}} \right\} \tag{35}$$

When $\rho_0(x_0) \simeq \overline{\rho_0}(x_0)$ — $x_0 \ll 1$ — the field is uniform, parallel to $\underset{\sim}{H_i}$ and of strength $H_i\,(\overline{\rho_0(x_0)}/\rho_i)^{2/3}$, essentially the result $H \propto \rho^{2/3}$. Just beyond R_0, although ρ_0 is dropping off exponentially to ρ_i, the mean density $\overline{\rho_0(x_0)}$ is still much larger than ρ_i, so that

$$\left|\frac{H_r}{\cos\theta}\right| \gg \left|\frac{H_\theta}{\sin\theta}\right|; \tag{36}$$

except near the equator, the field is nearly radial. Far enough away, when $\overline{\rho_0} \simeq \rho_i$ also, $\underset{\sim}{H} \simeq \underset{\sim}{H_i}$, corresponding to $x_0 \simeq x_i$. All these results are obvious geometrically from the essentially non-uniform dragging of initially straight field lines.

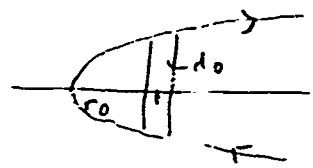
An external compression that has the high degree of symmetry required to yield (27) and (35) is not very plausible. However, a cloud approximating to this model could arise by fragmentation of a collapsing super-cloud with a large-scale field, but of a mass well above the limit (2:32) set by the virial theorem. In any case, if the magnetic field is to be able to decelerate the rotating cloud, it must have lines that traverse the cloud and extend into the medium surrounding. The model is about the simplest that one can think of, consistent with the freezing of the field.

We can now compute the magnetic force density $(\nabla \wedge \underset{\sim}{H}) \wedge \underset{\sim}{H}/4\pi$. The radial component opposes further spherical contraction. We may easily write down a condition that the (radial) gravitational force be everywhere large enough for at least approximately spherical collapse to continue; the condition is very similar to what we derive from the virial theorem for a cloud of radius R_0, density ρ_c and magnetic field $H_i\,(\rho_c/\rho_i)^{2/3}$. The significant feature is the θ- component of the magnetic force, which cannot be balanced by a spherical density field. This is particularly marked in the zone where the field lines are radial, so that the magnetic force reduces effectively to a simple magnetic pressure $-\partial(H_r^2)/8\pi r\partial\theta$, which must be non-zero since H_r by (35) varies strongly with θ. Thus immediately following the primary distortion (27) there is a secondary distortion of the density field, in which the magnetic forces build up for themselves a density field able to withstand the magnetic pinching. Since the field is frozen in, the secondary distortion of the density field also changes the magnetic field. Equilibrium is reached (for the zone of strong pinching) when the radial magnetic field has adjusted itself to be nearly uniform over the quadrant, with the density field likewise; but in a thin equatorial zone, a high density exerts a pressure able to balance the magnetic pinch:

$$\rho(\text{equator})\, c^2 \simeq \frac{H_r^{\;2}}{8\pi}, \tag{37}$$

c being the isothermal sound speed. The thickness of the zone is

$$d \simeq 2r_0 \left[\frac{\rho_0(r_0)}{\rho_{eq}(r_0)}\right]\left[\frac{\rho_0(r_0)}{\bar{\rho}_0(r_0)}\right]. \tag{38}$$

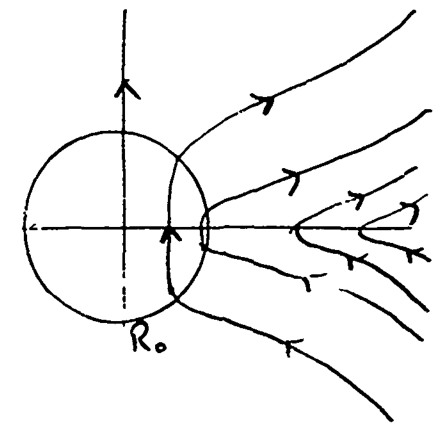


This follows because the zone is bounded above and below, at the radial distance r_0, by the critical field line which is changing over, from being perpendicular to the equator, to being nearly radial. Before the pinching, the height of the cylinder of unit cross section, that crosses the equator normally and is bounded by the critical field lines, is by (35)

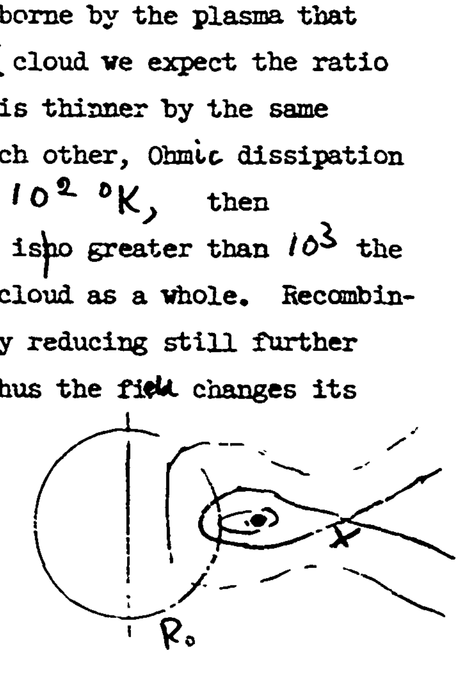
$$d_0 \simeq 2r_0 \left(\frac{\rho_0(r_0)}{\bar{\rho}_0(r_0)}\right); \tag{39}$$

after the pinching the height is reduced by the factor $\left[\rho_0(r_0)/\rho_{eq}(r_0)\right]$; whence (38). It is clear that the pinching will be strongest where the density is approaching the background value ρ_i; for H_r decreases like $1/r_0^2$ and $1/\bar{\rho}$ increases like r_0^2, while the density decreases like $\exp(-r_0/R_0)^2$.

The secondary distortion itself does not change the topology of the field - all the lines remain infinite. But the greatly increased curvature near the equator results in a sharp increase in the radial magnetic force at the equator, which now depends on the temperature of the gas, through c. (This shows up the limitations inherent in the virial and other integral theorems: they cannot take account of singular regions.) As the non-homologous contraction proceeds, both the radial distortion of the field and the secondary pinching get worse; even if initially the gravitational field were strong enough to overcome the increased magnetic resistance at the equator, in the subsequent flow the equatorial magnetic force in the pinched zone increases more rapidly than the gravitational, so that balance is achieved. Thus we see that if the field is strictly frozen in, isotropic contraction must break down. Gas flows down the approximately radial field lines towards the equator, so increasing the gravitational force density at the equator, and allowing further distortion of the field. Since the curvature of the field lines is towards the region of lower density, one would not expect hydromagnetic Rayleigh-Taylor instability to arise: exchanging field lines will not release energy (even if one felt certain about applying the energy test to this situation with its large mass motions).[30]



However, it is unlikely that the field will persist long as a highly distorted part of the local galactic field. First of all, the assumption that the bulk of the gas moves with the field becomes quite intenable once the pinched zone has been generated. We have already seen that when the magnetic force is "normal" - i.e. of order of magnitude $H^2/4\pi R_0$ — the drift that it forces the plasma to make relative to the neutrals is negligible compared with the bulk velocity, except when the plasma density becomes exceptionally low. But in the pinched zone the neutral gas drifts laterally under its enormous partial pressure. A new quasi-equilibrium is set up, with the neutral gas having a density roughly independent of θ and all the pressure of the radial magnetic field being borne by the plasma that (for the moment) is tied to the magnetic field. Since in an HI cloud we expect the ratio of plasma to neutrals to be $\sim 10^{-4}$, the new pinched zone is thinner by the same factor. But with oppositely directed field lines so close to each other, Ohmic dissipation can no longer be ignored. Provided the temperature stays near 10^2 °K, then rough numerical estimates show that even if ρ_c / ρ_i is no greater than 10^3 the Ohmic decay time is far shorter than the free-fall time for the cloud as a whole. Recombination of the plasma under the enormous compression will assist by reducing still further the amount of plasma available to keep the field lines apart. Thus the field changes its structure, first by forming a cusp-like neutral point, which then divides into an O-type and X-type point.



Field lines successively drift into X, where they divide into a finite part linked with the cloud, and an infinite part detached from it. The process slows up when the amount of magnetic flux over the quadrant through X is small enough for the pinching to be much reduced, so that Ohmic dissipation is cut down. Simultaneous with the disappearance of the pinched zone, the sharp curvature of the equatorial field lines is reduced, so that the radial magnetic force at the equator reverts in order of magnitude to the virial theorem prediction.

By this process, the field of the cloud has detached itself from the galactic field; its structure is more familiar, being similar to the vacuum field outside a dipole, superimposed on a weak background field. Since the magnetic energy outside the cloud is far greater than the thermal or gravitational energy, assuming normal densities, one tends to expect that in dynamical equilibrium the field will exert very weak forces - i.e. the current density perpendicular to $\underset{\sim}{H}$ will be much less than the order-of-magnitude estimate $(cH/4\pi R)$. For a purely poloidal field, this means that the field is nearly curl-free.[31] This argument, however, ignores the possibility that the strong field will itself build up, by compression, the thermal field required to balance it. It reverts to a field similar to the curl-free field only because by forming the pinched zone it brings about conditions in which the freezing of the field into the plasma is no longer valid.

In a primeval cloud of pure atomic hydrogen, with no external sources of ionizing radiation, the situation is a little more complicated because of the ionization of hydrogen by compression. However, we can show that the "Ohmic flash" in the pinched zone is again forced on the system. As the pinched zone approaches dynamical equilibrium, the drop in compressional energy generation causes excess recombination; the neutralised hydrogen drifts out, and the zone shrinks, compressional energy slowing up a little the recombination, until a new quasi-equilibrium is reached. Thus the distance between oppositely directed field lines systematically decreases - and just as in the HI cloud problem, at a rate determined not by the Ohmic diffusion, which is initially far too slow, but by the much more rapid drift of neutral atoms under their partial pressure. Ohmic diffusion is required to change the topology of the field: it occurs rapidly enough because the other process has forced the field lines together.

Many details still need study. It is not clear that the temperature in an HI cloud will stay near 100° K, for the Ohmic and compressional heat generated may be more than the radiators can cope with; the Ohmic resistivity may thus decline, and slow up the flash somewhat. However, the rate of heating can hardly become so low that the temperature drops back to 100° K: and since the Spitzer-Savedoff cooling time[2] is of the order of 100 years at normal densities and is much shorter at high densities, one feels fairly confident that the systematic snapping of field lines will occur once the cloud has begun its collapse. Further if the cloud is rotating about the axis of the field, centrifugal force will limit the amount of gas that can flow towards the cloud; and once a near-vacuum region has formed near the equator, the field has no difficulty in taking on the curl-free structure.

3.4 Summary of conclusions for the case with $\underset{\sim}{H}$ parallel to $\underset{\sim}{\Omega}$.

We have found

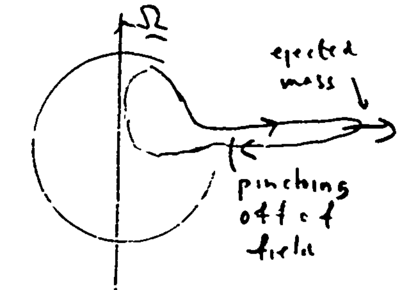
(a) if the cloud is freely falling, only those field lines which graze the cloud near the equator have a chance of efficiently decelerating the cloud by transport of angular momentum to infinity; for most field lines, the time of travel of hydromagnetic waves through the cloud is too long compared with the time in which free-fall increases the spin and the density:

(b) the distortion of the field when the cloud is formed leads to local breakdown in the freezing of the field, so that the cloud field systematically detaches itself from the local galactic field. The lines which graze the cloud near the equator are the first to detach themselves, so that they also soon cease to act as efficient transporters of angular momentum: the only lines which stay "infinite" in this symmetry are those that emerge near the poles, and so traverse the whole diameter of the cloud. It is unlikely that a more complicated initial field structure - e.g. a quadrupole-type- would alter these conclusions.

Thus during the collapse of the rotating magnetic cloud into a flattened structure, conservation of angular momentum is a good approximation. After the collapse the cloud can fragment into sub-systems with high orbital angular momentum, strong internal magnetic fields, and with centrifugal force of spin that is at first moderately small. But these sub-condensations in their turn will be limited in the degree of spherical collapse they can undergo: the conclusion that magnetic braking is not effective at this stage has left unsolved the problem as to how a mass of stellar order can reach the main sequence.

At sufficiently high densities the fragments become opaque, and the contraction is slowed up. The time-scale difficulty is thus relaxed. However, the detachment of the fragment's field from the cloud field is almost certain to persist, in spite of the fact that the opaque fragment is contracting more slowly than the free fall of the external gas; for the closed loops of field keep the local matter rotating with the fragment, and centrifugal force will prevent the neutral point from being dragged into the fragment.[32] Further, we have noted that meridian circulation within opaque bodies will tend to detach the internal field from the external. Thus this simple model of transport of angular momentum by hydromagnetic waves is not plausible during the Kelvin-Helmholtz stage also.

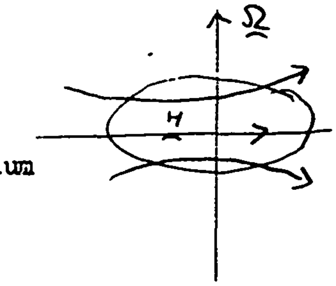
However, the physics of the problem is altered in one important aspect. When a body with a high internal pressure field flattens because of conservation of angular momentum, it tends to become rotationally unstable: the energy in the thermal field enables matter near the equator to spew out with the velocity of escape. This matter has more angular momentum than the average, so that the angular momentum per unit mass decreases a little because of this loss; but, as pointed out and exploited by Schatzman,[23] magnetic coupling between the emitted matter and the proto-star greatly increase the angular momentum carried off by a small amount of matter. Further, the spontaneous emission of matter automatically drags out part of the field that would otherwise be trapped by the internal circulation. (The elongated loop will exert strong pinching forces, and will ultimately detach itself from the internal stellar field). Thus a proto-star with a primeval internal field can contract, always staying on the brink of rotational instability; the amount of mass lost during the approach to the main sequence is much less than the associated loss of angular momentum. This picture is not adequate for stars later than Type F, which have angular velocities much lower than the more massive stars. Here Schatzman appeals to the analogue of solar activity that is presumably always associated with an extensive convection zone. Emission of matter from active surface zones will also transport a very high angular momentum-to-mass ratio, provided the matter expelled is magnetically coupled to the surface. Once the star has ceased to be on the verge of rotational instability, loss of mass through the equator ceases; but a magnetically active star will continue to lose angular momentum.



We thus have a very attractive possible explanation for the break in angular momentum at Type F: more massive stars reach the main sequence without passing through a regime with an extensive convective zone.

An ultimate aim of any theory of star formation is to predict the luminosity function. Our general conclusion is that the angular momentum problem must finally resolve itself during the slow contraction the main sequence, rather than during the earlier stages, when the temperature is still low and the cloud can flatten and fragment. Further research should therefore be directed towards estimating the masses of the final fragments in terms of the opacity, and to further elucidation of the Schatzman process during the slow contraction stage. In particular, one wants to be sure that the mass loss is small during the approach to the main sequence.

3.5 A cloud with $\underset{\sim}{H}$ and $\underset{\sim}{\Omega}$ perpendicular.



We now discuss briefly what we may expect to happen when the strong large-scale field is perpendicular to the angular momentum vector. The most striking contrast with the previous case is that a quasi-equilibrium state can be reached without the thermal pressure playing any role. In planes perpendicular to $\underset{\sim}{\Omega}$ the centrifugal force balances gravity, while a slight contraction parallel to $\underset{\sim}{\Omega}$ increases the magnetic force more rapidly than the gravitational; thus an initially spherical cloud achieves equilibrium with a spheroidal shape. But, equally, the cloud cannot begin to fragment. Excess flow down the field - required so that the density may increase more rapidly than $H^{3/2}$ - is prevented by centrifugal force; while flow parallel to $\underset{\sim}{\Omega}$ - so as to reduce the centrifugal parameter $\Omega^2/\pi G \rho$ - is prevented by the field.

Thus the evolution of such a cloud depends on the transport of angular momentum by the field. Since the cloud is in centrifugal balance, the magnetic transport does not have to try and keep pace with free-fall. On the contrary, the changes in the cloud occur at a rate determined by the angular momentum transport: the fact that free-fall has a time-scale somewhat shorter than magnetic braking means that close centrifugal balance is maintained at all stages.

The problem is difficult to treat in detail because of the absence of an axis of symmetry. Qualitatively one expects some transport of angular momentum from the cloud as a whole to the rest of the galaxy, so that the cloud contracts into a similar equilibrium state. But the distortion of the external field will again lead to detachment of the field lines from the galactic field and so to a cut-off in the angular momentum transport. Subsequent evolution must depend on the magnetic redistribution of angular momentum within a cloud. Since in the equilibrium state the angular velocity field will certainly not be uniform, twisting of the field will generate torque. A simple model has been studied,[34] using a bogus (axially symmetric) field maintained by a distribution of magnetic poles on the rotation axis. If the gas near the axis rotates uniformly, while that far from the axis lags, angular momentum is transferred out, so that the inner gas contracts and the outer gas moves further from the axis. As the mass distribution alters, so does the gravitational field, and the angular velocity field adjusts itself also; the subsequent angular momentum transport is also affected. Again, an exact theory must watch for the tendency of field lines to snap, and so break magnetic contact between regions of different rotation.

Qualitatively, then, one can see how systematically smaller blobs can arise - all in centrifugal-gravitational and magnetic-gravitational equilibrium. Once the blob becomes sufficiently opaque, then the energy of compression, generated in the contraction caused by magnetic transport of angular momentum, will heat up the blob, which will become rotationally unstable. It thus appears that again Schatzman's process is relevant to this case also: for it is an essential feature of the picture that the centrifugal force is important at all stages until the opacity becomes large, so that further strong magnetic braking is required if the blob is to become a main sequence star, and not just a set of the hypothetical planetary masses discussed in Chapter 2. Again, the distribution in mass of the globules at the time when they become opaque is probably a good approximation to the final luminosity function.

The effect of the field in preventing flattening towards a thermal disk state may have applications on the galactic level. We noted in Chapter 2 that unless one started with violent density variations, it was difficult to see how a rotating cloud could break up into a system with large z-motions. But if the collapse into a disk is halted by the transverse magnetic field, and sub-condensations form only because of magnetic redistribution of angular momentum, then perhaps the blobs will be able to break off from the magnetic field and fall only after their mutual collision cross-sections have been considerably reduced. There is thus a *prima facie* case for expecting there to be more kinetic energy retained in this case than when both $\underset{\sim}{H}$ and $\underset{\sim}{\Omega}$ are more or less parallel.

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